Self Control, Risk Aversion, and the Allais Paradox

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Introduction

- risk preferences and self-control problems are linked and should have a unified explanation
- choices made in the Allais paradox are a consequence of a self-control problem
- self-control can explain the results of recent experimental work by Benjamin, Brown and Shapiro [2006] on the effect of cognitive load on small-stakes risk aversion
- model based on Fudenberg and Levine [2006] of long-run versus short-run selves
- convex cost of self-control motivated by experiment of Shiv and Fedorikhin [1999]

Self-Control with a Cash Constraint

- a single long run patient self and sequence of short-run impulsive selves
- equivalently a single long-run agent who acts to maximize expected present value of per-period utility *u* net of self control costs *C*

$$U = \sum_{t=1}^{\infty} \delta^{t-1} \left[u(a_t, y_y) - C(a_t, y_t) \right]$$

a_t action chosen in period *t*

y_t state variable such as wealth

"opportunity-based cost of self control"

- *C* depends only on realized short-run utility and highest possible value of short-run utility
- latter called *temptation utility*

The Classical Problem of Hyperbolic Discounting

One candy bar today or two tomorrow

One candy bar a year from now or **two a year and a day from now**

In the latter choice choosing two a year and a day from now is a commitment that avoids the cost of self control

In the choice between today and tomorrow, there is no avoiding the internal conflict

The Bank and the Nightclub

infinite-lived consumer making savings decision.

periods $t = 1,2,\ldots$, LR discount factor δ

divided into two sub-periods: *bank* and *nightclub*

state $w \in \Re_+$ wealth at beginning of bank sub-period

bank subperiod consumption not possible

> wealth w_t divided between savings s_t, which remains at bank, and cash x_t which is carried to the nightclub

nightclub consumption $0 \le c_t \le x_t$with $x_t - c_t$ returned to bank at the end of period

$w_{t+1} = R(s_t + x_t - c_t)$, no borrowing, only income return on investment.

$u(c) = \log(c)$ where $\log(0) = -\infty$

reduced form preferences

consumption not possible in the bank so short-run self is indifferent

in the nightclub short-run self wishes to spend everything

$g(\overline{u} - u)$ cost of self-control, continuously differentiable, convex

temptation utility $\overline{u}$, realized utility is u

reduced form preferences for long-run self

$$U_{RF} = \sum_{t=1}^{\infty} \delta^{t-1} [\log(c_t) - g(\log(x_t) - \log(c_t))]$$

solution

no cost of self-control at bank

so choose optimal consumption without self-control costs

$$x_t = (1-\delta)w_t$$

then spend all pocket cash at nightclub: avoid all self-control costs

unanticipated decision at the nightclub

choice between two lotteries, A and B

largest possible loss less than agent’s pocket cash

short-run player in the nightclub simultaneously decides:

- lottery to pick
- how to spend the proceeds

self control cost

highest possible short-run utility from consuming all proceeds
temptation utility

$\max\{E\log(x_1 + \tilde{z}_1^A), E\log(x_1 + \tilde{z}_1^B)\}$

$\tilde{z}_1^j$ realization of lottery $j = A, B$

$\tilde{c}_1^j$ consumption chosen contingent on realization of lottery *j*

self-control cost

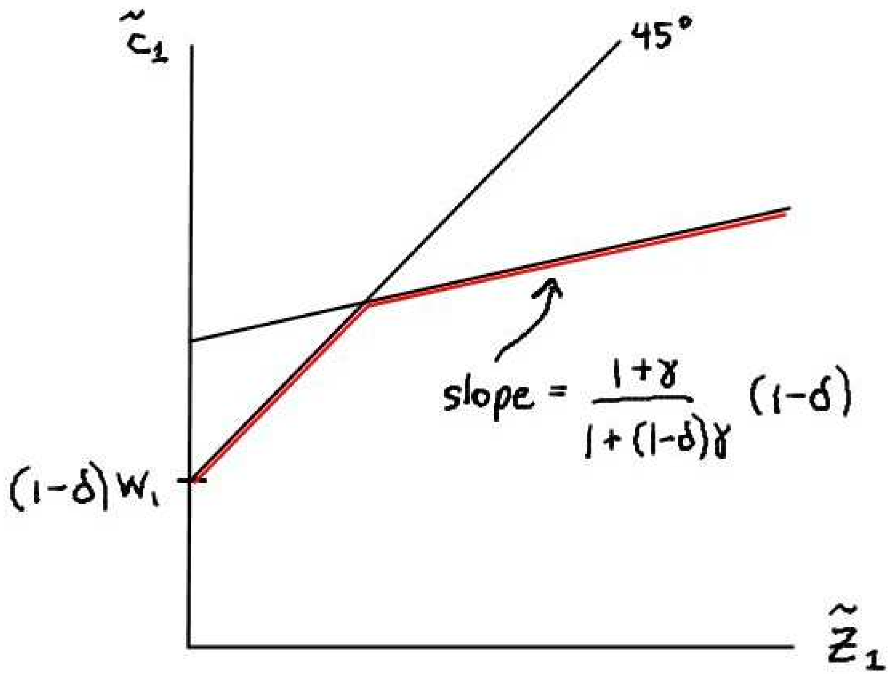
$$g\left[\max\{E\log(x_1 + \tilde{z}_1^A), E\log(x_1 + \tilde{z}_1^B)\} - E\log\left[\tilde{c}_1^j\right]\right]$$

overall objective of the long-run self

$$E\log(\tilde{c}_1) - \overline{g} + \frac{\delta}{(1-\delta)}E\log(w_1 + \tilde{z}_1 - \tilde{c}_1) + K$$

where is an irrelevant constant

Self-Control Solution



The Rabin Paradox

> “Suppose we knew a risk-averse person turns down 50-50 lose \$100/gain \$105 bets for any lifetime wealth level less than \$350,000, but knew nothing about the degree of her risk aversion for wealth levels above \$350,000. Then we know that from an initial wealth level of \$340,000 the person will turn down a 50-50 bet of losing \$4,000 and gaining \$635,670.”

The point being of course that many people will turn down the small bet, but no one would turn down the second. In our model, however, we can easily explain these facts, with, say, logarithmic utility.

too much risk aversion for small gambles

A : $(.5 : -100, .5 : 105)$

B : 0 chosen

	A	**B**
Utility	0.469	**0.630**
Gamma	2.079	**2.000**

add \$100 to all payoffs so there are no losses; still works

works for $a = .2, b = .15$ (but not for $a = .18, b = .13$)

works also if we set $b = 0$; doesn't need quadratic

Shiv Fedorikhin

- subjects asked to memorize two- or seven-digit number
- walk to table with choice of two desserts: chocolate cake, fruit salad
- pick a ticket for one desserts
- go to report the number and ticket in a second room
- seven-digit number chose cake 63% of the time
- two-digit number chose cake 41% of the time

our interpretation

- use of cognitive resources reduces those available for self-control
- cost of self-control is convex, so this increases marginal cost of self-control

further implications of convexity

replace the desserts with lotteries giving a probability of a dessert

utility difference between choices reduced

reduces marginal cost of self-control

so: fewer agents should give in to “temptation” of chocolate cake as the probability of winning a dessert is lower

change in ranking of lotteries as probability of winning the prize varies violated the independence axiom underlying expected utility theory

when cost of self-control is convex objective function non-linear in the expected utility of the short-run self, so the objective function that is maximized is not linear in probabilities, that is, the theory is not an expected utility theory.

Has the experiment been run?

Same idea very much as the Allais paradox

Kahneman and Tversky [1979] version

scenario one

A_1: $(.01 : 0, .66 : 2400, .33 : 2500)$

B_1 \$2400 for sure chosen

scenario two

A_2: $(.33 : 0, .34 : 2400, .33 : 2500)$ chosen

B_2: $(.32 : 0, .68 : 2400)$

the paradox

$$A_3 : (1/66 : 0, 16/33 : 2400, 1/2 : 2500)$$

then

$$A_1 = .66A_3 + .34B_1$$

$$B_1 = .66B_1 + .34B_1$$

independence axiom: choice between A_1 and B_1 same as choice between A_3 and B_1

$$A_2 = .68A_3 + .32 * 0$$

$$B_2 = .68B_1 + .32 * 0$$

independence axiom: choice between A_2 and B_2 same as that between A_3 and B_1

paradox arises from fact that choices differ

Calibration

Quadratic cost

$$g(\tau) = a\tau + (1/2)b\tau^2$$

use an iterative procedure to find unique solution of FOC

$$\gamma = a$$

$$c_1^* = \frac{(1-\delta)(1+\gamma)(w_1+z_1)}{\delta+(1+\gamma)(1-\delta)}$$

$$\tilde{c}_1 = \min\{c_1^*, x_1 + \tilde{z}_1)$$

$$\gamma = g'(\max\{E\log(x_1+\tilde{z}_1^A), E\log(x_1+\tilde{z}_1^B)\} - E\log[\,\tilde{c}_1^j\,])$$

The Allais Paradox

pocket cash x_1 is \$300

initial wealth w_1 is \$300,000

since $x_1 = (1-\delta)w_1$ corresponds to $\delta = 0.999$

take (obviously by fitting the data) $a = 2, b = 1.5$

	A_1	B_1
Utility	4.663	**4.667**
Gamma	3.169	**3.153**

Removing the $.66$ chance of 2400 reduces the temptation; if we choose the quadratic b large enough (1.5) we get a reversal

	A_2	B_2
Utility	**3.510**	3.509
Gamma	**2.874**	2.858

larger self-control parameters even $a = 2.2$ "safe" option chosen, so no paradox not everyone exhibits the paradox for these payoffs

not claiming everyone has parameters $a = 2.0, b = 1.5$ just that this is somewhere in the middle of the population distribution

original Allais paradox

A_1 : $(.01 : 0, .89 : 1{,}000{,}000, .1 : 5{,}000{,}000)$

B_1 : 1,000,000 for sure chosen

A_2 : $(.90 : 0, .10 : 5{,}000{,}000)$ chosen

B_2 : $(.89 : 0, .11 : 1{,}000{,}000)$

with logarithmic preferences B_1 never chosen for any reasonable wealth/pocket cash

does it makes sense to assume logarithmic preferences with respect to such large prizes?

modify utility function $u(5{,}000{,}000) = \log Y$

$1{,}206{,}000 \leq Y \leq 1{,}208{,}000$, $a = 2.0, b = 1.5$

optimal choices B_1 and A_2 consistent with the paradox

explanation of the paradox requires near indifference in both scenarios

"indifference" likely to be easier to achieve for thought experiments than for actual ones

Magnitude of Self-Control Cost

What does $a = 2, b = 1.5$ mean?

optimal levels of consumption for various winning amounts

(choice is between a certain gain and certainty of no gain)

Winning amount	Level of consumption
$0	$300
$105	$405
$500	$800
$1000	$925
$2500	$929
$10,000	$952
$50,000	$1075

Cognitive Load

experiment by Benjamin, Brown and Shapiro [2006]

Chilean high school juniors

made choices about uncertain outcomes

no cognitive load versus remembering seven digit number

B : safe option 250 pesos

A : risky option 50% chance of winning X , 50% of 0

fraction of subjects who choose the risky option B as a function of X.

“X”	No load	Cognitive Load
200	1/15	1/22
350	4/15	8/22
500	6/14	9/22
650	9/13	5/21
800	10/13	8/21

real, not hypothetical choices

subjects paid in cash at the end of session

1 $US= 625 pesos

weekly allowance was around 10,000

from this they had to buy themselves lunch twice a week usual experimental error/heterogeneity

some subjects choosing risky option even when expected value less than that of the sure thing

interesting aspect change

actuarially fair X=500 where risk aversion says choose A

prize of X=650

no cognitive load, many switch to the risky B

with cognitive load, switching is other way

we can’t explain the decline

our interpretation: no load, and the prize is increased to 650, some subjects switch to the risky alternative

do not switch when they are under cognitive load (treat switching back as measurement error) explanation:

risky alternative of 650 has a greater self-control problem than the certain alternative of 250

as the cognitive load increases, marginal cost of self-control goes up, so alternative is less likely to be chosen

No change

B: 50-50 randomization between 200 and 300 pesos.

“X”	No load	Cognitive Load
200	2/13	3/22
350	0/15	2/22
500	4/14	7/22
650	11/15	15/22
800	13/15	19/22

similar when no load, but they also switch when there is cognitive load explanation: A is less attractive due to risk, and the self-control cost associated with it is higher, so cognitive load has less effect

pocket cash to be 400 pesos

(with logarithm and 1000 pesos pocket cash no one would ever choose A no matter the self-control)

self-control parameters $a = 1.3, b = 0.65$

(if we use $a = 2.0, b = 1.5$ as before, then when X=650 option A is chosen)

note though: we are just asserting that both $a = 1.3, b = 0.65$ and $a = 2.0, b = 1.5$ are somewhere in the population distribution in both cases – no reason to think the marginal person is the same in both cases cognitive load increases marginal cost of self-control

we assume that this moves the parameter a from

$a = 1.3$ to $a = 1.4$

with the safe alternative B

for the lower parameter A (risky) is chosen; for the higher parameter B is chosen

with the risky B then A is always chosen regardless of the parameter

Token Donation Paradox

Number of tokens donated to the “common” in a public good contribution game (Isaac and Walker)

Fraction donating more than 0	Fraction donating more than 1/3	Fraction of possible tokens donated
0.23	0.10	0.07
0.58	0.33	0.29
0.55	0.30	0.24

Fehr-Schmidt

$$U_i(c_i) = c_i - \alpha \max\{c_j - c_i, 0\} - \beta \max\{c_i - c_j\}$$

non-linear in consumption

does not respect the risk preference of either player

makes a difference over what period of time consumption is measured

- this round or the entire session?

Dual Self

Assume short-run self has utility $\log(c_i) + \alpha \log(c_j)$

- preserves risk preference of both
- relevant period of consumption: time frame of short-run self
- leads to a "preference for fairness" based on changes in marginal utility due to relevance of pocket cash

altruism of short-run self predicts difference between "named" and "statistical" life

care only about lives SR can see, plus non-linearity

case 1: you can pay to save a you see life

case 2: you can pay to reduce the probability a life is lost (that you might see)

self-control problem greater in the former