Chapter 3.

The Model

3.1. Overview and Characteristics of the Model

Overview We present an auction-based asset market model to analyze price behavior in response to a shock to an asset and examine how the existence of circuit breakers makes a difference in price behavior. In this model, risk-neutral traders who maximize the expected payoffs make a bidding decision based on a private signal which is positively correlated with the unknown value of a shock. In addition to a fundamental shock which affects the future dividend stream of the asset, traders are faced with another source of uncertainty, a supply (or demand) shock which represents order imbalances at a particular date. Assuming that there is no further shock, prices as a function of traders' bidding strategy depend not only on the fundamental shock but also on a random draw of the supply shock. Whereas a large price change is more likely to be driven by a fundamental shock, a large realization of a supply shock can also bring about a large price swing. Since traders cannot distinguish one shock from the other, they make a Bayesian inference about the true value of the asset based on their beliefs about the distribution generating the supply shock as well as their own private signal.

Prices in this auction model are given as an order statistic and the clearinghouse executes the transaction at this price. Since prices are partially revealing due to the supply shock, traders update their beliefs about the true value of an asset using price information when available. In order to analyze the consequences of this updating of beliefs, we consider two consecutive rounds of price determination following shocks to the asset. There are two types of traders, sophisticated and naive traders, who show different behavior in updating their bids. Whereas sophisticated traders make a bidding decision by fully utilizing all the available information, naive traders with limited ability to process information summarize the multi-dimensional information vector into a single dimension. That is, naive traders respond to price information by adjusting their private signal to reflect their updated beliefs about a shock and make a bidding decision based on the adjusted signal. Since the adjusted signal is not a sufficient statistic for the available information, there inevitably incurs an information loss.

When there are no circuit breakers, markets clear in each round and price information is released as a single point. Since the convex combination of the two points gives a value between these two points, the adjusted signals of naive traders do not affect the price determined in the first round. Under this circumstance, updated bids of sophisticated traders based on this price information also result in the same price as determined in the first round since it is already consistent with their beliefs.

On the other hand, a triggering of the (upper) circuit breaker bound provides price information in the form of a truncated distribution. Belief adjustment based on the truncated price information causes traders to hold more optimistic beliefs about the true value of the shock. The adjusted signals of naive traders will reflect their optimistic beliefs and therefore result in greater updated bids. While the updated bids of naive traders place an upward pressure on prices, sophisticated traders behave conservatively since they know that the price will become greater than the equilibrium level due to aggressive bidding by naive traders. That is, they submit bids which are smaller than their initial bids. The market clearing price which results from the bidding prices of both types of traders is shown to overshoot the equilibrium level. However, it eventually converges to the equilibrium level as further rounds of trading follow.

While we assumed an once and for all shock to focus on the psychological effect brought by the presence of circuit breakers, we also discuss their presumed benefits on the assumption that supply shocks impinge on the market each period.[7] A large volume shock in the first round can lead to a deviation of prices from the equilibrium level determined by the fundamentals. However, as more realizations of supply shocks are observed in successive rounds of auctions, traders can accurately calculate the true value of the asset and prices eventually approach their equilibrium level. In this situation, the presence of circuit breakers may be beneficial by preventing a sudden price change due to a temporary volume shock. A release of information about the order imbalances while circuit breakers are in effect may help traders to recognize that price change is mostly due to a particular realization of a supply shock. Also, circuit breakers can affect a realization of the supply shock in the second round if they help to induce more value traders into the market.

However, we cannot sure which way it will go. A triggering of circuit breakers may scare traders away from the market rather than reassuring them, making a realization of the supply shock move in the opposite direction. Also, price overshooting occurs even under this circumstance if there are some traders who bid aggressively due to a triggering of circuit breakers. After all, whether circuit breakers are effective in moderating price volatility depends on which effect dominates the other.

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[7]There are other arguments for circuit breakers such as limiting credit risks related to margin calls and also preventing bottlenecks due to the limited capacity of exchanges. While our model does not incorporate such possibilities, we will discuss those arguments in Chapter 7.

Characteristics of the Model Our model has several distinctive features which distinguish it from other studies. First, auction mechanisms are employed as a trading rule and show an explicit price formation process. Whereas most studies of stock market behavior analyze a market where market makers exist, this paper seeks to model stock trading in a market without market makers. Trading in exchanges where market makers or specialists do not exist is best described as an auction market.[8] A desirable feature of auction models is that they capture many of the details of real stock markets. For example, in a typical stock transaction for a listed stock, a buyer places a limit order, *i.e.*, he instructs his broker to obtain the most favorable possible terms of trade but not to pay more than the suggested price. He expects to acquire the security whenever his bid is greater than the prevailing price. In this procedure, traders must make a bidding decision in ignorance of execution prices. In a rational expectations, Walrasian setting, on the other hand, agents behave as if they know the prices or submit demand schedules contingent on prices.[9] Although auction models have limitations such as a restriction on the amount each buyer can acquire, they provide a convenient device to address the question of how prices are formed.

Second, differential information among individuals is used as a basic motive for trading in this model. In order for trading to take place, some disparities in preferences, endowments and beliefs among individuals are needed. Trading in actual stock markets results from a mixture of the above factors. In this model, the motives of traders other than differential information are suppressed as large price swings accompanied by huge trading volumes are more likely to be an informational

[8]Their micro-structure is quite different from U.S or British exchanges in the sense that there do not exist traders who take their own positions as a market maker or a specialist does. On Japanese stock exchanges, for example, the members, called *saitori or nakadachi,* do not take their own positions. They simply execute orders according to a certain set of auction rules. See for details Takagi (1989).
[9]Milgrom (1981) points out that most rational expectations equilibrium models are not models of price formation and naive mechanisms leading to such equilibria can be severely manipulable.

phenomenon. This paper emphasizes the belief adjustment process of heterogenously informed traders facing both fundamental and supply shocks. In this sense, our model can be distinguished from other studies which focus on the transmission process of order flows (Greenwald and Stein, 1991) or choice of trade timing (Subrahmaynam, 1993).

Third, naive (noise) traders as well as sophisticated (rational) traders are present in the model. A recurring assumption in economic theory is that all individuals are fully rational. Questions have, however, been raised as to whether fully rational agents and the resulting rational expectations equilibrium can properly reflect economic reality.[10] There have been roughly two approaches to modelling agents better suited to explain actual economic phenomena. The first (the Bounded Rationality approach) is to assume boundedly rational agents by peeling off some degree of rationality from all the individuals in the model.[11] The alternative (Noise Trader approach) is to introduce a certain portion of "irrational" agents while allowing the others to maintain their full rationality.[12] Shleifer and Summers (1990) give a review on the noise trader approach. After defining noise traders as those whose *opinions* and *trading patterns* are subject to systematic biases, they illustrate three advantages of this approach as follows:

The noise trader approach provides tractable and more plausible theoretical

[10]As shown in the 'no trade theorem', for example, fully rational traders are too sharp to trade solely based on differences in private information, which overrules common sense intuition. See Milgrom and Stokey (1982) for the no trade theorem. The logic underlying the no trade theorem is also well summarized in Sargent (1993).
[11]Among those who take this approach, Thomas Sargent states that "the rational expectations hypothesis has two key aspects, individual optimality and the mutual consistency of beliefs. We interpret a proposal to build models with 'bounded rational' agents as a call to retreat from the second piece of rational expectations by expelling rational agents from model environments..." See Sargent (1993) pp. 1-25.
[12]Kyle (1985) and De Long, Shleifer, Summers and Waldman (1990) are among those applying this approach.

models..., yields a more accurate description of financial markets..., and yields new and testable implications about asset prices.... It is absolutely *not true* that introducing a degree of irrationality of *some* investors into models of financial markets "eliminates all discipline and can explain anything".[13]

In this paper, the second approach is applied and it is assumed that there are some portion of traders whose behavior is not fully rational due to the limited ability to process information.

The fourth characteristic of the model relates to the information content of prices. The recurrent idea in the rational expectations equilibrium models is that prices are fully revealing. The information conveyed by the equilibrium price is superior to any private information in the sense that price information is a sufficient statistic for diverse private information. It results in the following well-known paradox. If prices are fully revealing, an individual's optimal demand is independent of his private signal. Then, how can the equilibrium price system aggregate the individual's diverse private information and how can prices fully reveal all the diverse information? This paradox can be resolved if the price system aggregates information only partially. The price is fully revealing when there is only one source of uncertainty, namely, regarding the true value of the shock. In a typical auction model, price is given as an order statistic which is not fully revealing. However, as the number of traders becomes large as in this model, price converges to the unknown true value of the auctioned object. (Milgrom, 1979) Unlike typical auction models, our model has an additional source of uncertainty, that is, a supply shock.[14] Since prices are partially revealing in this situation, price information no longer swamps the information contained in private signals and traders supplement their private signals with the price information in

[13]See Shleifer and Summers (1990) pp. 19-33. All italics and double quotation marks in the quoted paragraph are from their paper.
[14]See Huang and Litzenberger (1988) pp. 259-283 and also Sargent (1993) pp. 116-125.

making inferences about the unknown true value of a shock.

3.2. Framework

Market environment Consider a market where M *indivisible* shares of an asset are traded.[15] Each share pays a liquidating dividend at a known time in the future. The price of the asset is subject to change due to exogenous shocks to the asset which affect the future dividends stream. M *indivisible* shares are traded in a simple clearinghouse market by $n > M$ traders. There is a single share constraint so that each agent can obtain a maximum of one share. At any time in the market, therefore, there are M shareholders and $(n-M)$ non-shareholders. The number of shares M is an unknown random variable due to a supply shock. This assumption reflects the fact that the number of buyers and sellers participating in stock trading at a particular date varies over time.

Traders are assumed to be risk-neutral and make trading decisions based on differential information about the true value of a shock. There are two types of traders: sophisticated traders (denoted S) and naive traders (denoted N). Naive traders are present in the model as a proportion α of n traders and sophisticated traders as a proportion $(1-\alpha)$. Trading behavior of each type is described later in this section.

Information structure Suppose there is a certain shock to the asset. The true value of the shock is unknown to the traders and denoted by V. Since we are focusing on

[15]Some kinds of circuit breakers like 'trading halts' on the NYSE are triggered when an overall market index hits the predetermined limit. To incorporate such cases, 'an asset' can be interpreted as a market portfolio.

price responses to one shock, we assume that successive shocks will come only after the trading procedures to resolve the effect of one shock have completed. People have a common prior on the distribution generating shocks. Its probability density function $\xi(\cdot)$ is given as follows:

$$\xi(v) = N(\mu, r_v) \tag{3.1}$$

where the precision parameter r_v is the reciprocal of the variance of V.

The traders, having access to different information sources, have different guesses about how much the shock to the asset is objectively worth. Each trader observes a real-valued random signal X_i in connection with the occurrence of a shock. Each private signal is treated as a random draw from a normal distribution with an unknown mean V and a (conditional) precision r_x. That is,

$$X_i = V + \varepsilon_i \qquad \text{where } \varepsilon_i \sim N(0, r_x), \tag{3.2}$$

We denote the conditional density of X by $f(\cdot)$.

Since price changes can be driven by a supply (or demand) shock as well as by a fundamental shock, we incorporate the possibility of a price decline due to a supply shock into the model by assuming that the number of shares M is a random variable. For example, a realization of large value of M indicates that there are more sellers than buyers in the market and vice versa. Since price is given as a order statistic in auction models, the M^{th} (highest) order statistic among n signals, denoted by $X^n_{(M)}$, plays an important role in the model. The distribution of $X^n_{(M)}$ depends not only on private signals but also on an exogenous random process governing the supply shock. Let us assume that M takes a value of m_k with a probability q_k where $k = 1,2,\dots,K$, $K \le n$ and $\sum_{k=1}^{K} q_k = 1$. Also define $m_k/n \equiv p_k$ and θ_k to be the $p_k{}^{th}$ quantile of *p.d.f.* of X. We borrow the following lemma about the asymptotic distribution of $X^n_{(m_k)}$, the $m_k{}^{th}$ order statistic among n signals.[16]

Lemma 1: Given the assumption on X in (3.2), $X^n_{(m_k)}$ is asymptotically distributed as a normal distribution with mean θ_k and variance $\sigma_{kk} \equiv q_k(1-q_k)/n[f(\theta_k)]^2$.

From the above lemma, it can be shown that a random variable $X^n_{(M)}$ is distributed with a mean $\sum_{k=1}^{K} q_k \cdot \theta_k$ and variance $\sum_{k=1}^{K} q_k^2 \sigma_{kk} + 2\sum_{k=1}^{K-1}\sum_{j=k+1}^{K} q_k q_j \sigma_{kj}$. Notice that θ_k can be expressed as $V + \lambda_k$ where λ_k is a fixed constant since θ_k is a given quantile of $f(\cdot)$ which has a mean V. Hence, $\sum_{k=1}^{K} q_k \cdot \theta_k = V + \lambda$ where $\lambda \equiv \sum_{k=1}^{K} q_k \cdot \lambda_k$. For simplifying calculation and notation, let us denote $Y = X^n_{(M)} - \lambda$. Then, Y is given as follows:

$$Y = V + \varepsilon_y \quad \text{where } \varepsilon_y \sim (0,\ r_y) \qquad (3.3)$$

where r_y, a precision of ε_y, is equal to $(\sum_{k=1}^{K} q_k^2 \sigma_{kk} + 2\sum_{k=1}^{K-1}\sum_{j=k+1}^{K} q_k q_j \sigma_{kj})^{-1}$ and ε_i and ε_y are independent of each other. Since Y is a linear transformation of an order statistic, it follows that X_i and Y are independent conditional on V and also that Y has the monotone likelihood ratio property (MLRP).[17] We interpret Y as a market signal in

[16] Lemma 1 can be found in Mood, Graybill and Boes (1974), p. 257.

[17] MLRP is defined as follows: Y has the (strict) MLRP if the likelihood ratio function $f(y|v)/f(y|v')$ is nonincreasing (decreasing) in y whenever $v'>v$ and nondecreasing (increasing) whenever $v>v'$. This definition is from Milgrom (1981). Milgrom also provides a proof that an order statistic among n random variables has the MLRP if they are independent and identically distributed.

the sense that the price is a bid submitted by the M^{th} highest signal holder. Whenever prices are known to traders, they deduce a market signal Y from price information and update their beliefs about V based on (3.3).

The above information structure summarized by (3.1) to (3.3) suggests that large values for some of the variables make the other variables more likely to be large than small. For example, a buyer whose valuation is high will expect that the true value is more likely to be high and believe that others have a high valuation also. This approximates the reality of stock markets. When a shock of great significance to the asset has occurred, it is more likely that people have a signal which indicates that something substantial has happened.

Trading mechanism The trading mechanism we have in mind is a computerized exchange rather than a dealership market where market makers or specialists exist. In those exchanges, stock trading is conducted according to two types of auction methods: *a call auction* or *a continuous auction*. A call auction method is used to establish opening prices at the beginning of each day (more specifically, each session). It places all orders received during some specified period of time preceding the opening of trading and sets the opening price so as to clear the market. We assume that trading takes place once in a period and follows a call auction method. Using auction jargon, the trading rule in this model can be identified as a *common-value sealed-bid double auction*.[18]

See Milgrom (1981), pp. 925-929.

[18]It is natural to take the common-value assumption since the auctioned asset has a single objective market value to all traders once the true value of a shock is known. We use a sealed-bid auction rather than an open outcry model since the call auction method we are assuming is typically based on sealed bids. Also, stock markets are basically a double-sided market where buyers and sellers coexist. The major difference distinguishing this model from auction models is the fact that the number of shares is a random variable while auction models assume the number of auctioned objects as fixed and known to all traders. See McAfee and McMillan (1987) for a survey of the auction literature.

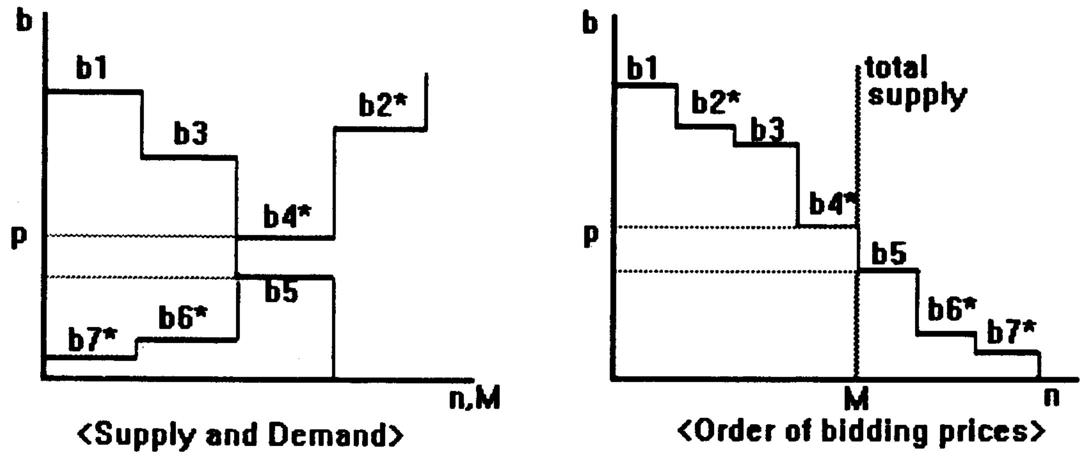
Trading proceeds as follows. Having received a private signal X with the occurrence of a shock, each trader submits a single "limit order" to the clearinghouse. For a shareholder, this order is an offer to sell his share at any price which is equal to or greater than his asking price. For a non-shareholder, it is a maximum bid below which he is willing to buy one share. The clearinghouse in the model plays the role of an auctioneer (like a computer in a computerized exchange system). It receives all bidding and asking orders from n traders, determines a market clearing price and executes a transaction under that price.

In order to find a market clearing price, the clearinghouse obtains the market supply schedule $S(p)$ by arranging asking orders by M shareholders in an ascending order and the demand schedule $D(p)$ by arranging $(n-M)$ non-shareholders' orders in a descending order. The market clears at a price where $D(p)$ and $S(p)$ intersect. There is a continuum of prices at which the market clears. For computational convenience, the market clearing price p^* is set at the highest intersection of $D(p)$ and $S(p)$, that is, $p^* = \sup\ \{p : D(p) = S(p)\}$. Alternatively, the clearinghouse arranges all bids in a descending order and find the M^{th} highest bid as a market clearing price.[19] We follow the latter method since it helps to find the equilibrium price more easily. The equivalence of the two methods is proven in the Appendix and intuitively described in Figure 3.1. At this price, orders are executed and the M highest bidders become the shareholders for the next period.

Whereas trading proceeds as described above when there are no circuit breakers, the existence of circuit breakers may keep prices from fully adjusting to the market clearing price in a round of the auction. Since the upper and lower limit up to which prices can change in a round are specified by circuit breakers, the determination

[19]This market clearing mechanism can be found in Friedman and Aoki (1992).

Figure 3.1: Market Clearing**

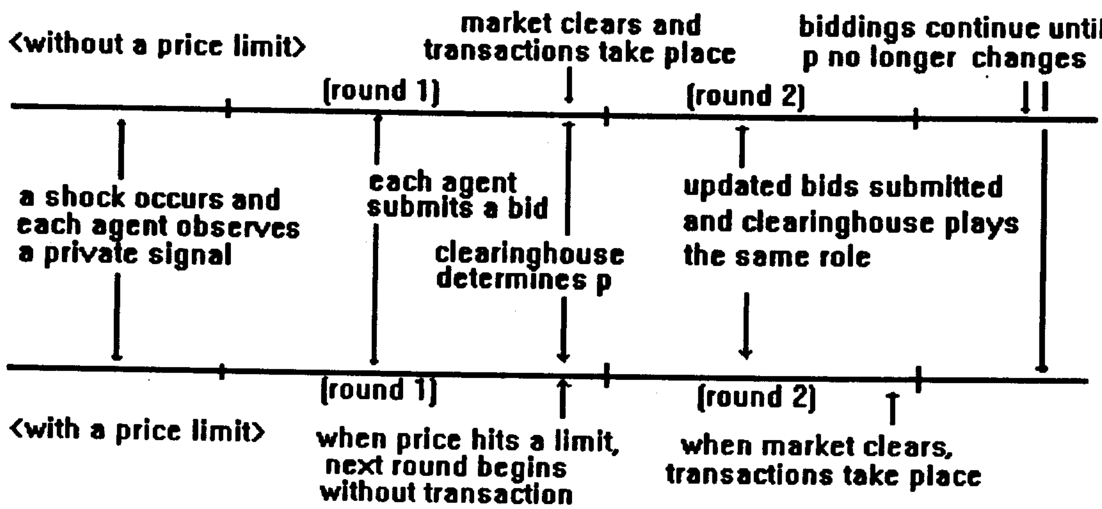


* denotes the seller ** when n=7 and M=4

of the market clearing price in response to an unexpected large shock may require repeated call auctions over several days. When there is a market excess demand at the first round (assuming a positive shock), *i.e.*, there are more than M traders who submit their bids at the upper limit, the clearinghouse announces that the market is not cleared and begins the second round of bidding without executing transactions at the upper limit.[20] The bidding procedure continues until traders with the M highest bids can be identified. When the market clears, the traders who have submitted higher (lower) bids than the market clearing price become the shareholders (non-shareholders). The

[20]Trading stops as soon as the price hits a predetermined limit in case of the 'trading halts' type of circuit breakers and no transaction takes place at the limit price although there are traders who are willing to trade even at the limit price. For example, in the New York Stock Exchange (NYSE), trading in all stocks is halted for one hour when the Dow Jones Industrial Average (DJIA) declines 250 points from the previous day's close, and for two hours when DJIA declines 400 points. See the Fact Book of the NYSE (1992). On the other hand, in the case of price limits, transactions take place between buyers and sellers who are willing to trade at the limit price. Although some shareholders (non-shareholders) become non-shareholders (shareholders) in the next period due to transactions at the limit price, such a change in traders' identities does not affect their bidding decisions.

Figure 3.2: Temporal Illustration of Trading Process



temporal illustration of the trading mechanisms with and without circuit breakers is provided in Figure 3.2.

__*Behavior of traders*__ The objective of traders is to maximize the current expectation of final payoffs. A buyer acquires a share and pays the market clearing price p if his bid b is greater than p. On the other hand, a seller sells his share at p in case his bid is lower than p and keeps his share otherwise. Hence, when trader i tenders a bid b and the price is realized to be p, his payoff denoted by $U(V, X_i, b)$ is given as follows:

$$U(V,X_i,b) = \begin{cases} (V-p)\cdot 1_{\{p\le b\}} & \textit{for buyer } i \\ V\cdot 1_{\{p\le b\}} + p\cdot 1_{\{p>b\}} & \textit{for seller } i \end{cases} \tag{3.4}$$

where $1_{\{e\}}$ is an indicator function, which takes the value one if the event e occurs and zero otherwise. Knowing that prices are partially revealing in this model, they supplement private information with price information whenever available.

There is a difference in the bidding behavior of the two types of traders. The sophisticated traders who are fully rational understand the entire model structure including the fact that there exists a portion α of naive traders. They also recognize that price determined in the auction is the M^{th} highest bid among n bids and make a strategic bidding decision considering how the others behave. Their strategies constitute a Nash equilibrium and are consistent with their beliefs.

On the other hand, the naive traders are assumed to have limited analytical capacity. First, they do not consider the strategic interaction among traders and take price as exogenously given.[21] Whereas sophisticated traders recognize the possibility that when everyone evolves (*e.g.*, adopts aggressive bidding strategies) price can change correspondingly, naive traders regard price as an exogenous function of Y, the market opinion on which they do not have an influence. Let us define a function mapping Y into p by $\phi : Y \to p$. Then, price is perceived as $p = \phi(Y)$ for naive traders.

The second assumption about the naiveté is related to their belief adjustment process. Having less capability to process information, naive traders adopt a relatively simple learning and adaptation strategy. They respond to the newly available information by adjusting their *signals* using the adjustment parameter γ, $0 < \gamma < 1$, which represents how much importance naive traders put on their own signal in updating beliefs about V.[22] In this game, information regarding the true value of a

[21]This assumption ignores the possibility that each trader's current bidding decision may affect other agents' contingent behavior and thus affect his own future trading opportunities. Friedman (1991) adopted this assumption and called it a "Game against Nature."

[22]The extreme case that $\gamma \approx 1$ implies that each trader considers his own private signal only. When $\gamma \approx 0$, on the other hand, people ignore their own signal completely. It is most probable that $0 < \gamma < 1$. When γ is such that $E[V \mid X = x, Y^*] = E[V \mid X' = \gamma x + (1-\gamma)Y^*]$, the adjusted signal x' well represents their reservation price given the available information.

shock is revealed at more than one stage. In the beginning of the first round when price information is not available, a trader makes a bidding decision solely based on his private signal x. When he obtains price information at the end of the first round, revealing that the best estimate of market signal Y is equal to Y^*, he adjusts his private signal x to $x' = \gamma \cdot x + (1-\gamma) \cdot Y^*$ and makes a bidding decision for the next round based on the new signal x'. That is, naive traders summarize a multi-dimensional information vector (private and price information) into a single dimension. When they have a proper adjustment parameter reflecting each variable's precision, the new signal x' may reflect the reservation value of V given the available information. However, since the adjusted signal is not a sufficient statistic for both private and price information, it inevitably entails ignoring information that would be useful in calculating the optimal decision. Such an information loss may cause them to make a mistake in their bidding decision.

When there is no circuit breaker, price information is given as a single point, that is, $Y^* = y$. The adjusted signal $x' = \gamma \cdot x + (1-\gamma) \cdot y$ summarizes the updated belief about V. Notice that x' is smaller than y for $x<y$ and greater than y for $x>y$. Even if they make a bidding decision based on the adjusted signal, it does not affect the price. The updated bids based on x' well represent their reservation price and result in the same price as determined in the first round.

However, this relationship breaks down when there are circuit breakers. A limit-triggering event results in price information in the form of a truncated distribution that the market clearing price is greater than the upper limit $\overline{\delta}$, that is, Y is greater than c where $c \equiv \phi^{-1}(\overline{\delta})$. Given this price information, the best forecast of Y is $E[Y|x, Y \geq c]$ and naive traders adjust their signal $x' = \gamma \cdot x + (1-\gamma) \cdot E[Y|x, Y \geq c]$. Although the proper γ makes the adjusted signal adequately represent their reservation price so that $E[V|x, Y \geq c] = E[V|x']$, their bids based on x' are not optimal from the perspective of sophisticated traders. Suppose that all traders submit bids based on x'. Since x' is greater than x for all x unlike the case without circuit breakers, it results in aggressive bidding causing prices to rise. If they are rational enough to recognize that a greater price occurs due to this aggressive bidding, they can deduce that the true y is smaller than $\phi^{-1}(p)$ and find that what they wanted to pay was more than what they are willing to pay. That is, $E[E[V|x']|Y = y] = E[V|x', y] > E[V|x, y]$. However, naive traders who regard price as exogenous given do not consider such a possibility and believe that a greater price is due to a greater Y. Consequently, their naiveté results in irrationally aggressive bids that put upward pressure on prices. The behavior of sophisticated and naive traders are summarized in Table 3.1.

Table 3.1 Behavior of Sophisticated and Naive Traders

	sophisticated	naive
price	the M^{th} highest bid $p = B_M$	an exogenous function of Y $p = \phi(Y)$
belief adjustment	Bayesian updating $E[V|x, Y^*]$	signal adjustment using γ $E[V|x'],\ x' = \gamma x + (1-\gamma)Y^*$
bidding strategy	$b_S = b_S(x, Y^*)$	$b_N = b_N(x')$

3.3. A Benchmark: The Case without Circuit Breakers

When there are no circuit breakers, traders can bid whatever price they want. The market clears in each round at the price of the M^{th} highest bid B_M. Any buyer (seller) who submits a bid higher (lower) than the market clearing price buys (sells) a share. Other buyers and sellers do not transact.

Before analyzing traders' strategies, we make a simplifying assumption that there are sufficiently large number of traders so that they ignore the difference between the M^{th} and the $(M+1)^{th}$ order signal. In this situation, it can be shown that the optimal bidding price for the buyer is the same as the one for the seller if and only if they have the same private signal.[23] This assumption not only allows us to analyze the strategy of one side of traders, but also offers an advantage that an ordering by bidding prices of traders is equivalent to an ordering by their private signals.

Since the bidding strategy of a sophisticated trader is different from a naive trader, we first analyze the sophisticated trader's strategy. (when all traders are rational $(\alpha = 0)$) We also see how it brings a different result if traders are naive $(\alpha = 1)$ and finally analyze the general case in which both traders are present.

Equilibrium with the Sophisticated Traders We identify competitive behavior with (non-cooperative) symmetric Nash equilibrium behavior. A pure strategy for a trader is a function converting his information into a bid. Let us denote the strategy of sophisticated trader i who has a signal $X_i = x$ by $b_{S,i}(x)$. Holding the other traders'

[23]Although this assumption reflects an aspect of real stock markets, it is made to simplify the analysis. The double auction imposes considerable difficulties in formalizing tractable models because the strategy of a buyer is different from that of a seller even if they have the same signal. However, as the number of traders becomes sufficiently large, the magnitude by which an individual trader can affect the price becomes trivial and the strategic differences between a buyer and seller vanish. A proof is provided in Appendix 2.

strategies as fixed, trader i may regard the M^{th} highest bid B_M as a random variable. His strategy $b_{S,i}$ is called an optimal response to the opposing strategies if

$$b_{S,i} \in \arg\max_b \ E[U(V, X_i, b) | X_i = x] \tag{3.5}$$

If each $b_{S,i}$ in an *n-tuple* $(b_{S,1}, b_{S,2}, \cdots)$ is an optimal response to the other strategies, it is called an equilibrium point.

Let us define a function $\varphi(x,y) = E[V| X_i = x, Y = y]$, which is increasing in both arguments since X_i and Y have the (strict) monotone likelihood ratio property. Since traders are assumed to be risk-neutral, $\varphi(x,y)$ is the reservation price for trader i if he were able to observe $Y = y$. For example, buyer i would be willing to pay any price less than $\varphi(x,y)$ to acquire a share but would not do so at any higher price.

Theorem 1: Let $b_S(x) = \varphi(x,x)$. Then the n-tuple of strategies $(b_S, b_S, \cdots, b_S)$ is an equilibrium point in a market without a price limit.[24]

Proof: Let us show that the optimal bidding strategy of trader i, as a solution to (3.5), is equal to $b_S(x)$ when all the other traders follow the strategy $b_S(x)$. Since $b_S(x)$ is increasing in x, traders with higher private signals tend to submit higher bidding prices at equilibrium. Since the market clearing price is the M^{th} highest bid submitted by the M^{th} highest signal holder, it follows that $p = B_M = b_S(Y)$. Then trader i's maximization problem is as follows:

[24]This derivation of the equilibrium strategy follows the one used in Milgrom and Weber (1982). Whereas there is a fixed number of buyers in their model since they analyze the typical one-sided auction market, the number of the auctioned object M is a random variable and also both buyers and sellers are present in this model.

$$\begin{aligned}
\underset{b}{Max} \quad & E[\ (V-p)\cdot 1_{\{p\le b\}}\ |\ X_i = x] \\
&= E[(V-B_M)\cdot 1_{\{B_M\le b\}}\ |\ X_i = x] \\
&= E[\ E[(V-B_M)\cdot 1_{\{B_M\le b\}}\ |\ X_i,Y]\ |\ X_i = x] \\
&= E[\ E[(V-b_S(Y))\cdot 1_{\{(b_s(Y)\le b\}}\ |\ X_i,Y]\ |\ X_i = x] \\
&= E[\ (E[V|X_i = x,Y]-E[V|Y,Y])\cdot 1_{\{b_s(Y)\le b\}}\ |\ X_i = x] \\
&= \int_{-\infty}^{b_s^{-1}(b)} \{(\varphi(x,\omega)-\varphi(\omega,\omega)\}\ h_{Y/X}(\omega)\ d\omega
\end{aligned} \tag{3.6}$$

where $h_{Y/X}(\cdot)$ is a conditional density of Y given x. The second equality comes from the law of iterated expectations. The maximum is achieved by integrating over $\{\omega\colon \varphi(x,\omega)-\varphi(\omega,\omega)\ge 0\}$. Since $\varphi(x,\omega)-\varphi(\omega,\omega)$ is positive for $\omega<x$ and negative for $\omega>x$, (3.6) is maximized when $b_s^{-1}(b)=x$. Hence, $b=b_S(x)$. *Q.E.D.*

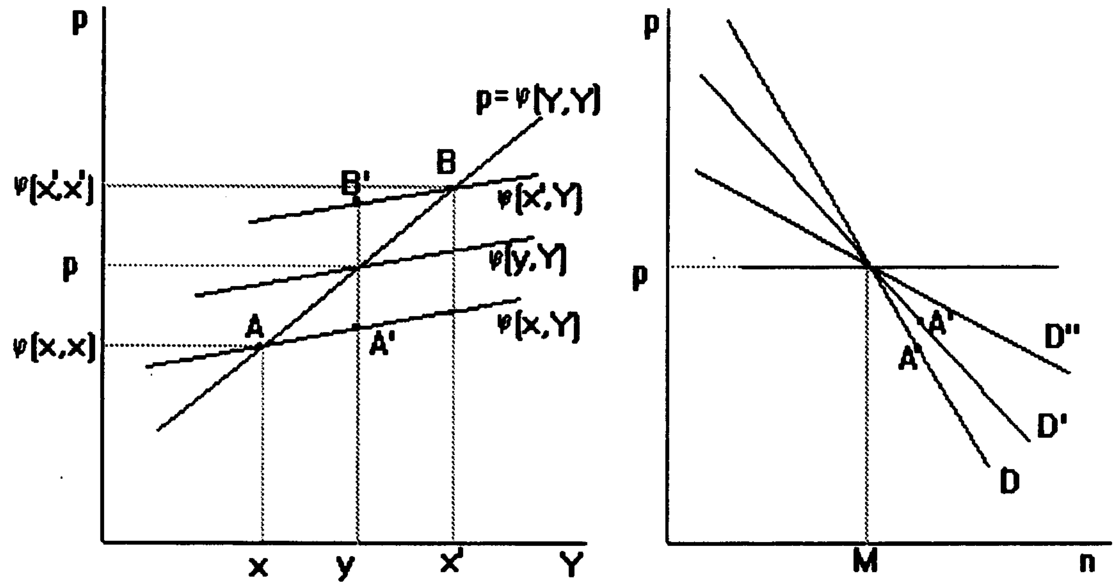
Theorem 1 states that it is optimal for each trader to submit his reservation price as if Y were equal to his own signal x. The optimal strategy $b_S(x)$ can be explained by Figure 3.3. Remember that $\varphi(x,y)$ is the reservation price of a trader given that his signal is x and $Y=y$. Since Y is an unknown random variable until the market is cleared, $\varphi(x,Y)$ denotes his reservation price function. The optimal bid for a trader who has a signal $x<y$ is indicated by point A in the figure. The strategy $\varphi(x,x)$ guarantees that whenever the market clearing price p is greater than his bid, his reservation price conditional on p is smaller than p. He sells (does not acquire) a share if he is a shareholder (non-shareholder), which is what he wants to do at p. The same logic applies to a trader with $x>y$. Since price is the M^{th} highest bid,

$$p=\varphi(Y,Y) \tag{3.7}$$[25]

---

[25]When ε_y in (3.3) follows a normal distribution, (3.7) is equal to $p=\dfrac{r_v\cdot\mu+(r_x+r_y)\cdot Y}{r_v+r_x+r_y}$.

Figure 3.3: Price Determination in the Benchmark Model



When the market clearing price in the first round is known to all, traders update their beliefs about the true value of the shock using price information and tender a new bid for the next round. We prove in the Appendix, however, that the updated bids do not change the price. Consider again Figure 3.3. After Y is realized to be equal to y, a trader with the initial signal x will submit a new bid $\varphi(x,y)$ for the next round, which is indicated by A' in the figure. Notice that A' is greater than A but still smaller than p. Also, the updated bid for a trader with $x' > y$, indicated by B', is smaller than his initial bid but still higher than p. The updated bid stays the same for a trader whose initial bid is equal to p. The right-hand side of Figure 3.3, showing the market demand schedule, describes that updated bids do not change the market clearing price although the slope of the schedule changes.[26] Hence, the price given in (3.7) remains the same for successive rounds unless further shocks arrive.

Bidding Strategy of the Naive Traders Since naive traders regard price as $p = \phi(Y)$, we require explicit knowledge about ϕ in order to analyze their strategy. We shall impose a restriction on ϕ so that ϕ is consistent with their understanding of the information structure. Suppose a trader's bid is realized to be equal to the market clearing price p. Then, he would believe that the average market opinion is the same as his belief about the true value of a shock and submit the same bid for the next period. That is, his reservation value of the shock conditional on p would be equal to p. On the other hand, any trader who submits a bid lower (higher) than p would think that his signal is smaller (larger) than the market player's signal. Hence, the price function ϕ is restricted to satisfy the following:

$$E[V \mid X_i = Y, p = \phi(Y)] = \phi(Y) \text{ and increasing in } X_i \tag{3.8}$$

Let $\phi(Y) = \varphi(Y,Y)$. Then, it can be shown that $\phi(Y)$ is a unique function satisfying (3.8).[27] However, the qualitative results of the paper do not change due to the choice of ϕ and $\phi(Y)$ can be understood as a normalization.

Let us denote a bidding strategy of the naive traders by b_N. The optimal bid of naive trader i who has a private signal $X_i = x$ is a solution to the following:

---

[26]In the figure, D'' is drawn to be more elastic than D'. The precision of Y assumed in D'' is greater than the one assumed in D'. It indicates that the slope of market demand schedule based on the updated bids becomes more elastic as traders believe that market information is more informative than their private information. In the extreme case when $r_x / r_y \cong 0$, the schedule becomes horizontal at the market clearing price.

[27]A proof is provided in Appendix 4.

$$b_{N,i} \in \arg\max_{b} E[(V - \phi(Y)) \cdot 1_{\{b \geq \phi(Y)\}} | X_i = x] \tag{3.9}$$

Theorem 2: Suppose that naive traders' beliefs on price is given as $\phi(Y) = \varphi(Y,Y)$. Then, the optimal bid of naive trader i whose signal $X_i = x$ is $\varphi(x,x)$.

Proof: Since $p = \phi(Y) = \varphi(Y,Y)$, the maximization problem for trader i is as follows:

$$\begin{aligned}
&\underset{b}{Max}\ E[(V - \phi(Y)) \cdot 1_{\{b \leq \phi(Y)\}} \mid X_i = x] \\
&= E[\, E[(V - \phi(Y) \cdot 1_{\{b \leq \phi(Y)\}} \mid X_i, Y] \mid X_i = x] \\
&= \int_{-\infty}^{\phi^{-1}(b)} \{\varphi(x,\omega) - \varphi(\omega,\omega)\}\ h(\omega / x)\ d\omega
\end{aligned}$$

The maximum is achieved when $\phi^{-1}(b) = x$. Hence, the optimal bid for naive trader i is equal to $b_{N,i} = \phi(x)$. *Q.E.D.*

Notice that the optimal bid for a naive trader is the same as that of a sophisticated trader as long as they have the same private signal. However, it no longer holds when there are circuit breakers as will be shown later.

The market clearing price as the M^{th} highest bid is equal to $\varphi(Y,Y)$, which is the same as in (3.7). After the market clearing price is known, naive traders adjust their signal x into $x' = \gamma \cdot x + (1-\gamma) \cdot y$ where $y \equiv \phi^{-1}(p)$ and make a bidding decision based on x' for next period.[28] However, the updated bids based on these new signals do not change the equilibrium price since the updated signal x' is a convex combination of x and y which takes on a value between x and y. An updated bid by a trader whose

[28]When ε_y follows a normal distribution and $\gamma = r_x/(r_x + r_y)$, it can be shown that $\varphi(x,y) = \phi(x')$.

private signal is smaller (bigger) than y is still smaller (bigger) than the market clearing price determined in the first round. Hence, the market clearing price remains the same.

Equilibrium with Both Types of Traders When both types of traders are present in the model ($0 < \alpha < 1$), sophisticated traders behave strategically knowing how naive traders behave. They treat the naive traders' strategy as given. The equilibrium strategy of the sophisticated traders and the resulting market clearing price depend on the portion α of the naive traders. The sophisticated trader i's problem is given as follows:

$$\underset{b}{Max}\ E[(V-p)\cdot 1_{(b\geq p)}\ |X_i = x]$$

subject to (3.10)

$$b_N(x) = \varphi(x,x)$$

$$n\cdot\{\alpha\cdot(1-G_N(p))+(1-\alpha)\cdot(1-G_S(p))\} = M$$

where $G_N(\cdot)$ and $G_S(\cdot)$ is the cumulative distribution function of the bidding prices for each type of traders. The last equation in (3.10) represents the market clearing condition that the number of bids higher than the market clearing price is equal to the number of shares offered for sale. The optimal bidding strategy and the resulting equilibrium price must satisfy the above three equations simultaneously since the market clearing depends on the strategy of sophisticated traders whose bidding decisions in turn are based on the price being determined.

Suppose that the sophisticated traders except i follow the strategy $b_S(x) = \varphi(x,x)$. Since the market clearing price is again equal to $\varphi(Y,Y)$ given the naive trader's bidding strategy $\varphi(x,x)$, the maximization problem for sophisticated trader i becomes the same one as given in (3.6). Hence, it is optimal for him to submit

$\varphi(x, x)$, which results in the same market clearing price as in (3.7).

In the second round, traders update their bids using price information. However, the updated bids do not change the market clearing price p determined in the first round since the updated bids submitted by traders whose initial bids were lower (greater) than p are still smaller (greater) than p. As far as the market clearing in the first round provides price information as a signal point, which is the case when there is no price limit, the presence of naive traders does not change the market clearing price determined in the first round.

3.4. The Existence of Circuit Breakers and Price Overshooting

When there are circuit breakers, traders should choose their bids within the prespecified upper and lower price limit. Since the possible limit-triggering provides further information on the true value of the shock, their strategy in presence of circuit breakers might differ from the strategy without circuit breakers. Throughout the analysis, we assume that the market is cleared in the second round for simplicity. Since we assumed that transactions do not take place until the market is cleared, the optimal strategy for the first round is the same as the one without circuit breakers except that traders with greater or smaller bids than the limit price should submit the upper or lower limit bid. Hence, we focus on the bidding strategy of traders for the second round.

Suppose that the upper limit $\overline{\delta}^1$ is triggered in the first round. This limit-triggering provides information that the market clearing price is equal to or greater than the upper limit. From the information $p \geq \overline{\delta}^1$, they can deduce that Y is greater than c. If we define $f(Y|v)$ to be the conditional probability density function of Y, then people have the following belief regarding $f(Y|v)$ after the first round.

$$f(Y|v) = \begin{cases} 0 & \text{for} \quad Y < -c \\ \int_{-\infty}^{-c} f(Y|v)\,dy & \text{for} \quad Y = -c \\ f(Y|v) & \text{for} \quad -c < Y < c \\ \int_{c}^{\infty} f(Y|v)\,dy & \text{for} \quad Y = c \\ 0 & \text{for} \quad Y > c \end{cases} \qquad (3.11)$$

When people update their beliefs about V using (3.11), they have greater reservation prices for the asset. That is,[29]

$$E[V|X_i = x,\ Y \geq c] > E[V|X_i = x,\ Y] \qquad (3.12)$$

Based on the information given in (3.11), traders make a bidding decision for the second round. We first analyze how rational traders respond to the price information provided by limit-triggering. In the following, we denote the bidding strategies of traders when there is a price limit by $\bar{b}_S$ and $\bar{b}_N$ to distinguish them from the case without circuit breakers. (An upper bar in the notation indicates the case with a price limit and we delete the superscript in traders' bidding strategy denoting the second round since we focus on the second round.)

Equilibrium with the Sophisticated Traders Given the market information $Y \geq c$, an

[29]This inequality can be proven using the monotone likelihood ratio property. For every nondegenerate prior distribution ξ on v and every y and y' in the range of Y with $y'>y$, the posterior distribution $\xi(v|Y=y')$ dominates $\xi(v|Y=y)$ in the sense of first-order stochastic dominance. Since higher values of V are integrated with a higher density, the posterior mean takes on a greater value.

optimal bidding strategy $\bar{b}_{S,i}$ of sophisticated trader i in the second round is given as

$$\bar{b}_{S,i} \in \underset{\underline{\delta}^2 \le b \le \bar{\delta}^2}{\arg\max}\, E[\, U(V, X_i, b) \mid X_i = x,\ Y \ge c] \tag{3.13}$$

Although traders have greater reservation prices due to the limit-triggered event, the equilibrium price is the same as the one determined in a market without circuit breakers, as shown in the following theorem.

Theorem 3: Let $\bar{b}_S(x) = \begin{cases} \bar{\delta}^2 & \text{if} \quad \varphi(x,x) \ge \bar{\delta}^2 \\ \varphi(x,x) & \text{if} \quad \underline{\delta}^2 < \varphi(x,x) < \bar{\delta}^2 \\ \underline{\delta}^2 & \text{if} \quad \varphi(x,x) \le \underline{\delta}^2 \end{cases}$ (3.13)

Then the n-tuple of strategies $(\bar{b}_S, \bar{b}_S, \ldots, \bar{b}_S)$ *is an equilibrium point in a market where there is a price limit.*

Proof: Suppose all other traders follow the strategy in (3.13). Since $\bar{b}_S(Y) = \varphi(Y,Y)$ when the market is cleared, $\bar{p} = B_M = \varphi(Y,Y)$. Trader i's maximization problem is given as follows:

$$\begin{aligned}
&\underset{\underline{\delta}^2 \le b \le \bar{\delta}^2}{Max}\ E[(V - \bar{p}) \cdot 1_{\{\bar{p} \le b\}} \mid X_i = x,\ Y \ge c] \\
&= E[\, E[(V - B_M) \cdot 1_{\{B_M \le b\}} \mid X_i, Y] \mid X_i = x,\ Y \ge c] \\
&= E[\, E[(V - \bar{b}_S(Y)) \cdot 1_{\{\bar{b}_S(Y) \le b\}} \mid X_i, Y] \mid X_i = x,\ Y \ge c] \qquad (3.14) \\
&= E[\ (E[V \mid X_i = x, Y] - E[V \mid Y, Y]) \cdot 1_{\{\bar{b}_S(Y) \le b\}} \mid X_i = x,\ Y \ge c] \\
&= \int_c^{\bar{b}_S^{-1}(b)} \{\varphi(x,\omega) - \varphi(\omega,\omega)\}\, h(\omega / x, \omega \ge c)\, d\omega
\end{aligned}$$

where $h(\omega / x, \omega \ge c)$ is a conditional density of Y given $X_i = x$ and $Y \ge c$. Since $\varphi(x,\omega) - \varphi(\omega,\omega)$ is monotonically decreasing in ω and zero for $\omega = x$, the maximum is achieved when $\bar{b}_S^{-1}(b) = x$. Suppose trader i's signal x is smaller than c. Since (3.14) is always negative, it is optimal for him to submit the lowest bid allowed by the stock exchange, that is, $b = \underline{\delta}^2$. When $x \geq c$, on the other hand, the maximum can be found by integrating until $\bar{b}_S^{-1}(b) = x$. Hence, $\bar{b}_{S_i} = \varphi(x,x)$. If $\varphi(x,x) \geq \bar{\delta}^2$, he should submit $\bar{\delta}^2$ which is the maximum bid available in the second period. When $\underline{\delta}^2 < \varphi(x,x) < \bar{\delta}^2$, the optimal strategy is to submit $\varphi(x,x)$. Hence, $\bar{b}_S$ is the optimal strategy for trader i. *Q.E.D.*

Notice that the optimal bidding strategy $\bar{b}_S(x)$ is equivalent to the strategy $b_S(x)$ for the case without circuit breakers, except that bids greater or lower than the limit price are transformed into the upper or lower limit bid. Although the price information provided by the limit triggering affects those whose initial bids are smaller than the lower limit $\underline{\delta}^2$, their updated bids are still lower than $\underline{\delta}^2$. Hence, they should submit the lowest bid $\underline{\delta}^2$. On the other hand, those whose initial bids are greater than $\underline{\delta}^2$ will find it optimal to behave as if they ignore the market information provided by the limit-triggered event. This is because a market clearing in the second round will provide price information as a point which dominates the previous range information, $Y \geq c$.

To see why, suppose that traders bid more aggressively due to limit-triggered event. Then, the market clearing price p' will be greater than p given in (3.7). In this situation, a trader who has submitted p' as his bid will realize that he is the M^{th} highest signal holder and will regret his aggressive bidding since his reservation value $\varphi(y,y)$ is smaller than his bid p'. Cleverly recognizing the sequential nature of this game, sophisticated traders follow the same strategy as before. As a result, the market clearing price is the same as the one determined in a case without circuit breakers.

Behavior of the Naive Traders Given the price information $p \geq \overline{\delta}^1$, naive traders adjust their signal into $x' = \gamma \cdot x + (1-\gamma) \cdot E[Y|x,\ Y \geq c]$ where $c \equiv \phi^{-1}(\overline{\delta}^1)$ and make a bidding decision based on x'. Since $E[Y|x,\ Y \geq c]$ is greater than x for all x, the adjusted signal is greater than the initial signal for all traders. That is, the information $Y \geq c$ provided due to limit-triggering causes them to hold more optimistic beliefs about the true value of a shock. Given x', the optimal bid of naive trader i for the second round is a solution to the following:

$$\overline{b}_{N,i} \in \underset{\underline{\delta}^2 \leq b \leq \overline{\delta}^2}{\arg\max}\ E[(V - \phi(Y)) \cdot 1_{\{b \geq \phi(Y)\}} |\ X_i = x'] \tag{3.15}$$

Compare (3.15) to the optimal bidding strategy of sophisticated traders given in (3.13). Whereas sophisticated traders fully utilize the available information $X = x$ and $Y \geq c$, naive traders' bidding decision is based on the adjusted signal x'. Since the maximization problem in (3.15) is equivalent to (3.9) except a change in signal, the optimal bid $\overline{b}_N$ for naive trader i as a solution to (3.15) can be shown to be as follows:

$$\overline{b}_N(x) = \begin{cases} \overline{\delta}^2 & if \quad \varphi(x',x') \geq \overline{\delta}^2 \\ \varphi(x',x') & if \ \ \underline{\delta}^2 < \varphi(x',x') < \overline{\delta}^2 \\ \underline{\delta}^2 & if \quad \varphi(x',x') \leq \underline{\delta}^2 \end{cases} \qquad (3.16)^{30}$$

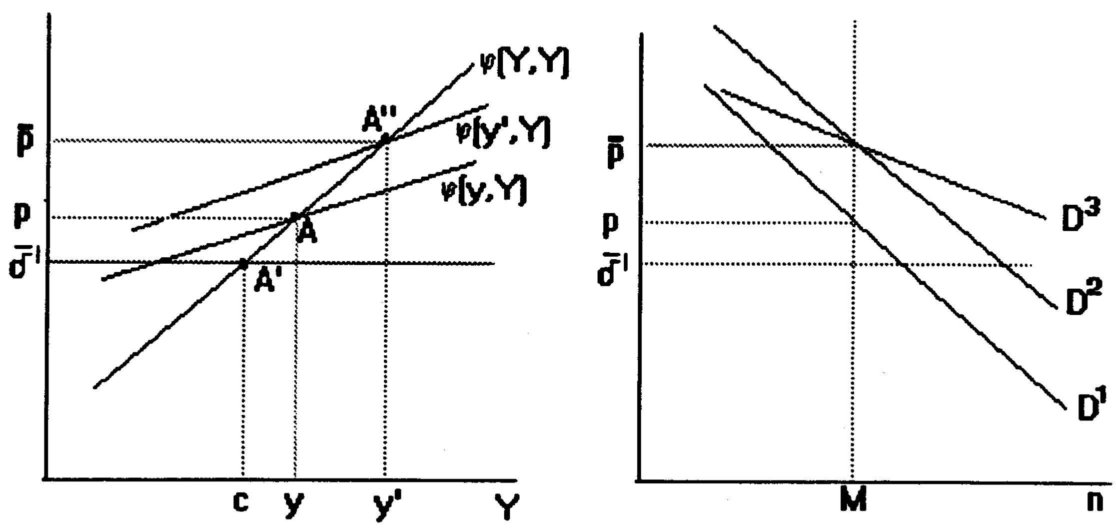
Since $\varphi(x',x')$ is greater than $\varphi(x,x)$, the market clearing price $\varphi(y',y')$ is greater than the one determined in a market without circuit breakers. After the market is cleared in the second round, traders adjust their signal x' into $x'' = \gamma \cdot x' + (1-\gamma) \cdot y'$ where y' is $y' = \phi^{-1}(\overline{p})$. However, the updated bids based on x'' do not affect the price

[30] A proof is given in Appendix 5.

determined in the second round since the updated signal y'' of the M^{th} highest signal holder stays the same as y'.

The behavior of naive traders is explained in Figure 3.4. Suppose a trader whose signal is equal to y, the M^{th} highest among n signals. The reservation price functions based on his initial and updated signal are drawn as $\varphi(y,Y)$ and $\varphi(y',Y)$. In the first round, he should submit $\overline{\delta}^1$, indicated by point A', although he wants bid higher. A triggering of the upper limit shifts his reservation price function upward. An updated bid A'' in the second round is greater than A which would have been submitted without circuit breakers. The figure on the right describes the market demand schedules. The market demand schedule D^1 in the first round shifts into D^2 reflecting the change in traders' beliefs due to the limit-triggered event. We also see that the schedule D^3 for the third round does not change the price determined in the

Figure 3.4: Naive Trader's Bidding Strategy



second round although its slope changes due to the adjustment of beliefs after market clearing.

Notice that this adjustment behavior results in price overshooting when there are circuit breakers. Compared to the case when the price information is released as a point, the limit-triggered event provides price information in the form of a truncated distribution. Whereas the adjusted signal takes a value between their own signal and market signal Y in a case without circuit breakers, the adjusted signals take greater values than their initial signals when circuit breakers are triggered. Signal adjustment reflecting more optimistic beliefs about V due to a limit-triggering makes traders bid more aggressively, and consequently the market clearing price $\overline{p}$ becomes greater than the price which would have been determined without circuit breakers.

<u>Equilibrium with Both Types of Traders</u> When both types of traders are present, sophisticated traders behave strategically knowing that naive traders follow the bidding strategy of (3.16). The maximization problem for sophisticated trader i is given as follows:

$$\max_{\underline{\delta}^2 \le b \le \overline{\delta}^2} E[(V-\overline{p})1_{(b\ge\overline{p})} | X = x, Y \ge c]$$

subject to (3.17)

$$\overline{b}_N(x) = \varphi(x', x')$$

$$n\cdot\{\alpha\cdot(1-G_N(\overline{p}))+(1-\alpha)\cdot(1-G_S(\overline{p}))\} = M$$

If we define $F(\cdot)$ as the cumulative distribution function of private signals, the market clearing condition in (3.17) becomes as follows:[31]

[31]Since $\overline{b}_N(\cdot)$ is a monotonically increasing function, $G_N(\overline{p}) = \text{Prob}(\overline{b}_N(X) \le \overline{p})$ $= \text{Prob}(X \le b_N^{-1}(\overline{p})) = F(b_N^{-1}(\overline{p}))$. It can also be shown that $G_S(\overline{p}) = F(b_S^{-1}(\overline{p}))$.

$$\alpha \cdot \{1 - F(\bar{b}_N^{-1}(\bar{p}))\} + (1-\alpha) \cdot \{1 - F(\bar{b}_S^{-1}(\bar{p}))\} - F(Y) = 0 \tag{3.18}$$

where $F(Y) = M/n$. From (3.18), we can define an implicit function $\bar{p} = r(\bar{b}_S, Y)$.[32] Any combination of $\bar{p}$, $\bar{b}_S$ and Y satisfying $\bar{p} = r(\bar{b}_S, Y)$ gives (3.18) the status of an identity. Given the sophisticated traders' strategy $\bar{b}_S$, the market clearing price can be expressed as a function of Y:

$$\bar{p} = \pi(Y) \tag{3.19}$$

where $\pi' > 0$. Given (3.19), the optimal bidding strategy of the sophisticated trader $\bar{b}_S$ is defined as follows:

$$\bar{b}_s \in \underset{\underline{\delta}^2 \le b \le \bar{\delta}^2}{\arg\max}\ E[(V - \pi(Y)) \cdot 1_{\{b \ge \pi(Y)\}} | X_i = x, Y \ge c] \tag{3.20}$$

The optimal strategy $\bar{b}_S$ as a solution to (3.20) is consistent with the market clearing condition since $\pi(Y)$ is a function satisfying (3.18). Since it is difficult to derive the optimal strategy of the sophisticated traders as an explicit function, we shall analyze the qualitative properties of the equilibrium.

The equilibrium of this game is summarized by the strategies of each type of trader, $\bar{b}_S$, $\bar{b}_N$, and the resulting market clearing price. The equilibrium strategy of the sophisticated trader should be the one which guarantees the condition that his bid is

[32]Let us denote (3.18) by $R(\bar{p}, \bar{b}_S, Y) = 0$. Since R has continuous partial derivatives $R_{\bar{p}}$, $R_{\bar{b}_s}$ and R_Y and also $R_{\bar{p}}$ is nonzero for every point of $\bar{p}$ $(R_{\bar{p}} = -\alpha g_N(\bar{p}) - (1-\alpha) g_N(\bar{p}) < 0)$, the condition under which the implicit function theorem holds are satisfied. This condition is sufficient for the existence of an implicit function. For the implicit function theorem, see Chiang (1974), pp. 216-227.

greater than the market clearing price p if and only if his reservation price on V is greater than p. Hence, for any value of x and Y, $\bar{b}_S$ should satisfy the following:

$$E[V|X_i = x, \bar{p} = \pi(Y)]\begin{cases} > \pi(Y) & \text{iff} \quad \bar{b}_S > \bar{p} \\ = \pi(Y) & \text{iff} \quad \bar{b}_S = \bar{p} \\ < \pi(Y) & \text{iff} \quad \bar{b}_S < \bar{p} \end{cases} \tag{3.21}$$

Lemma 2: Suppose that the price $\bar{p} = \pi(Y)$ is greater (smaller) than $\varphi(Y,Y)$. Then, it is optimal for sophisticated trader i to submit a bid which is smaller (greater) than $\varphi(x,x)$. When $\pi(Y) = \varphi(Y,Y)$, the optimal bid is equal to $\varphi(x,x)$.

A proof is provided in the Appendix. The above lemma can be explained as follows. Suppose that the market clearing price $\bar{p}$ is greater than $\varphi(Y,Y)$. If a sophisticated trader submits a bid $\varphi(x,x)$ and his bid happens to be equal to $\bar{p}$, he will realize that his reservation price $\varphi(x,y)$ is smaller than $\varphi(x,x)$ since $y = \pi^{-1}(\bar{p}) < \varphi^{-1}(\bar{p}) = x$. Recognizing this, he will find it optimal to submit a bid which is smaller than $\varphi(x,x)$.

Based on Lemma 2, we can characterize the equilibrium as follows.

Theorem 5: When both types of traders are present, the optimal bid of sophisticated trader i as a solution to (3.20) is smaller than $\varphi(x,x)$. Also, the market clearing price $\bar{p}$ determined in a market with circuit breakers is greater than p, the one determined in a market without circuit breakers.

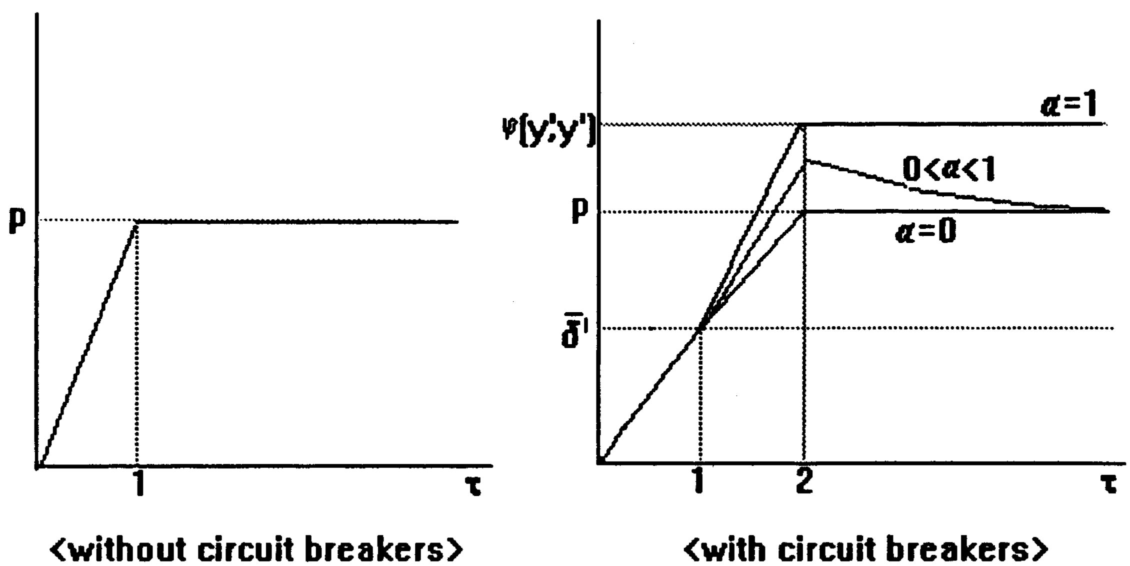
Proof: The market clearing price in (3.18) is an increasing function of both $\bar{b}_S$ and $\bar{b}_N$. Notice that $\bar{b}_N = \varphi(x',x') > \varphi(x,x)$ since $x' > x$. First, suppose $\bar{b}_S(x) \geq \varphi(x,x)$. Then the market clearing price is greater than the one determined in a market without a limit. Given that $\overline{p} > p$, the strategy $\overline{b}_S$ which is greater than $\varphi(x,x)$ is not optimal considering Lemma 2. Second, suppose that $\overline{b}_S(x) < \varphi(x,x)$ and $\overline{p} = p$. Given that $\overline{p} = p$, the optimal bid of the sophisticated trader should be equal to $\varphi(x,x)$, which contradicts $\overline{b}_S(x) < \varphi(x,x)$. Third, suppose that $\overline{b}_S(x) < \varphi(x,x)$ and $\overline{p} < p$. Given that $\overline{p} < p$, the optimal bid $\overline{b}_S(x)$ should be greater than $\varphi(x,x)$, which is a contradiction. Finally, when $\overline{b}_S(x) < \varphi(x,x)$ and $\overline{p} > p$, $\overline{b}_S$ is consistent with the resulting price $\overline{p}$ $(> p)$. *Q.E.D.*

The market clearing price overshoots the equilibrium level which would have been determined without circuit breakers. After the market is cleared in the second round, traders update their beliefs on V using price information. Since the sophisticated traders can deduce y from price information, their bidding price for the next round is equal to $\varphi(x,y)$. On the other hand, the naive traders deduce Y using $Y = \phi^{-1}(\overline{p})$ and adjust their signal x' into $x'' = \gamma x' + (1-\gamma)\phi^{-1}(\overline{p})$. Since we are focusing on price behavior, it is enough to only look at the bidding price of a naive trader whose initial signal is y. His updated signal for the third round is equal to $y'' = \gamma y' + (1-\gamma)\phi^{-1}(\overline{p})$. Since $\phi^{-1}(\overline{p}) < y'$, y'' is smaller than y' and the resulting price in the third round is smaller than the price in the second round. In subsequent rounds, his updated signal becomes smaller as $\overline{p}$ decreases and this process continues until the price reaches $\overline{p} = \phi(y)$. When $\overline{p} = \phi(y)$, his updated signal stays at y and the price does not change unless further shocks arrive.

Figure 3.5 summarizes the results. When there is no circuit breaker, the price immediately jumps to its equilibrium level, irrespective of the existence of naive traders. On the other hand, price overshoots its equilibrium level when there are circuit breakers. How much price overshoots depends on the portion α of naive traders and the precision of Y. When traders believe that Y is more informative than their own signal, the price information released by the limit-triggering event makes them bid higher. Also, when there are more people who make a bidding decision based on the optimistic beliefs due to the limit-triggering event, the magnitude of the price overshooting becomes greater.

We identify this price overshooting as an institution-induced phenomenon since it would not have occurred except for the presence of circuit breakers. When there is no limit on price movements, the market emits price signals as a single point. Since a convex combination of any two points gives a value between these two points, traders' updated bids based on price information do not cause prices to overshoot even with naive traders. On the other hand, the existence of circuit breakers provides price information as a truncated distribution. Belief adjustment based on the truncated price information causes some traders to hold more optimistic beliefs about the true value of

Figure 3.5: Comparision of Prices with and without Circuit Breakers



the shock. That is, the existence of circuit breakers itself becomes a source of panic trading by enticing otherwise 'well-behaved' traders to bid aggressively and consequently brings about price overshooting. In this situation, the existence of circuit breakers not only delays the incorporation of new information into prices, but also impairs the price discovery process.

3.5. Discussion

Throughout the analysis, we have examined the effect of circuit breakers under the simplifying assumption that there is no further shock until the effect of one shock is fully resolved. While we assumed a once and for all shock in order to focus on the psychological effect brought about by the presence of circuit breakers, shocks continuously impinging on actual stock markets. Under such circumstances, the arrival of a positive shock following a negative shock may partially offset the latter when there are circuit breakers. However, such offsetting due to opposing fundamental shocks does not seem to be the presumed benefits of circuit breakers suggested by their proponents. They are concerned the adverse effect of a large price swing caused by informationless panic trading. The potential benefits of circuit breakers can be incorporated into our model as follows.

To focus on large price changes which are not justified by fundamentals, let us assume that supply shocks arrive in each period whereas a fundamental shock comes only in the first round. In this model, successive supply shocks can be represented by realizations of a random variable M^{τ} in each period, where M^{τ} is a number of shares offered for sale at round τ.[33] A relatively large realization of M^τ indicates that there are more sellers than buyers and the M^{th} highest signal Y at round τ takes a lower value. Since traders submit their bids in ignorance of what supply shock is realized, prices depend on not only private signals on which their bids are based, but also on realizations of M^τ. For example, although traders follow the same strategy, different realizations of supply shock result in different prices.

Suppose that a large volume shock hits the market in the first round, that is, the number of shares M^1 offered for a sale at the first round takes on an extremely large value so that prices fall away from their equilibrium level which is determined by fundamentals. Since traders cannot tell whether the lower value of Y is due to a fundamental shock or a supply shock, they submit updated bids for the next round using their beliefs about the distribution of Y. As more buyers come to the market in the second round so that M^2 is realized as a moderate value, which is likely considering its randomness, the market clearing price becomes higher and approaches the equilibrium price. As auction procedures continue, traders get more accurate information about V from large number of observations of supply shocks and prices eventually converge to their equilibrium level. That is, a large, temporary volume shock can lead to a deviation of prices from their equilibrium level.

In this situation, the presence of circuit breakers may be beneficial in facilitating the price discovery process. First, release of information about order imbalances in the first round while circuit breakers are in effect can prevent large fluctuations of prices due to the supply shock. If information about the number of shares offered for a sale at the first round is released to traders, they can recognize that

[33]There is a possibility that supply shocks are functionally dependent on the fundamental shock and have the same sign. However, if a functional relationship between two shocks is known to traders, they will respond by adjusting their bids considering such a relationship.

price declines are mostly due to the supply shock. When they submit updated bids based on this information, price approaches the equilibrium level.[34] Second, if circuit breakers can restore investors confidence and induce more value buyers to the market, this results in a lower realization of M^2. Then, the resulting price in the second round will be closer to the equilibrium level.

However, it might be too optimistic to believe that the above positive feedback loop of circuit breaker mechanisms will be effective in reality. Since tradings are halted or limited during short periods of time (for example, one hour in case of 'trading halts' on the NYSE), there may not be enough time for exchanges to process and release information in time to get a response from the public. Even if information about order imbalances is available, it is hard to distinguish information-based trading from noise. Also, it may take longer time for the stock market's natural long-term investors to step in and take a position when the market is undervalued. They might feel safe by waiting and watching the market rather than reacting quickly.

In addition, a triggering of circuit breakers can scare traders away from the market rather than reassuring them. Then, the realization of M^2 would become even larger, causing price to drop further. Also, even under the assumption of continuous shocks, price overshooting may occur if there are some traders who bid aggressively due to a triggering of circuit breakers. Hence, whether circuit breakers are effective in moderating price volatility depends on which effect dominates the other. Although the benefits of circuit breakers seem to exceed the costs in the case of price changes due to a supply shock, it should also be noted that circuit breakers are blunt instruments which, once introduced, are triggered upon the prespecified price change regardless of

[34]As shown in a typical auction literature where a fixed number of auctioned objects is assumed, prices converge to the unknown true value as the number of bidders becomes large. See, for example, Milgrom (1979).

whether price changes are due to a fundamental or supply shock.