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Essays on Capital Markets: Frictions and Social Forces

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by

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For my Mother, my sister Stephanie, and Tessie.

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ABSTRACT OF THE DISSERTATION

Essays on Capital Markets: Frictions and Social Forces

by

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Doctor of Philosophy in Economics

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Professor David K. Levine, Committee Co-Chair

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Chapter One surveys the importance of social forces in introducing and implementing financial market structures in different environments. I discuss the ability of uncoordinated population behavior to implement or impact social conventions and government or bank opportunistic financial behavior. I emphasize the roles of information, the incentives that agents face, and the ability of agents responses to be effective.

Chapter Two investigates the stability of monetary exchange equilibria and the conditions necessary for monetary equilibrium to emerge endogenously. Previous research on money as a medium of exchange depended on initial or steady state conditions to select equilibria. Those approaches are unsatisfactory for addressing emergence, because the determining element is either exogenous or there are multiple equilibria without criteria for selecting among them. I construct an N-person non-cooperative anonymous game from an economy with many commodities and bilateral exchange. The environment is restricted so that in addition to direct barter opportunities there is a single commodity which can be used for intermediate exchanges. I use recent advances in evolutionary learning to characterize the conditions when monetary trade is typically observed over the long run regardless of initial conditions and discuss how informational assumptions affect transition time.

Chapter Three examines the role of uncertainty in determining interest rates in an economy with incomplete markets when uncertainty is partially common and partially unique to individuals. Insurance against individual endowment fluctuations must be arranged through trade with other individuals, by buying and selling assets. When there is aggregate uncertainty the ease with which persons in the economy can self-insure depend on how large their insurance needs are and how the values of the assets with which they must insure are correlated with their need to buy and sell them. Thus the rate of return of insuring assets will depend in a critical way on the exact relationship between aggregate and idiosyncratic risk in the economy. Following Levine (1993), I compute a Markov equilibrium of an economy with trembles that can be made arbitrarily close to the original. I find that increasing aggregate uncertainty increases rates of return and increasing the amount of idiosyncratic uncertainty decreases rates of return.

Chapter 1

The Role of Social Forces in Capital Markets

1.1 Introduction

This paper surveys the role of social forces in the introduction and stability of monetary systems. We explore the potential for sustainable fiat and commodity monetary systems in each of several different environments; government money, bank money, and no monetary authority at all. Aspects of this issue, in particular regulated and unregulated bank money, have been argued extensively by the opponents and proponents of the so-called free-banking school. Although there is a good deal of overlap, the focus here is narrower and rather than seeking a policy recommendation we seek to identify and characterize the social forces capable of enforcing and

innovating monetary systems.[1] In particular we hope to shed some light on questions such as the following.

Paper Currency in Persia. History records a variety of non-commodity currencies that have been used at various times and places: paper, stones, and shells. In 1294 AD the Persian king attempted to introduce a paper fiat currency. Despite a declared death penalty for those who refused to accept the currency as legal tender, the attempt collapsed within two months as traders deserted the bazaars (see Davies (1994) p.182). Paper fiat money had been introduced previously in China and was widely used there for a long time even as the printing press was used to extract significant seigniorage. Why was the introduction in Persia a failure?

The Great Debasement. Henry VIII financed a significant part of his extensive military expansion (and his lifestyle) through a series of debasements. The scale of the debasements of silver coinage was huge, the newer of two silver coins minted just ten years apart (1552 vs. 1542) had only 25% of the silver content of the older coin. Successful debasement calls for a large quantity of reminting (which is voluntary) to bring in seigniorage. Over the same period that silver was debased, gold coins were also debased, but to a much lesser extent and apparently generating much less

[1] Free-banking attacks the presumption that central banks are necessary. It proposes that central banks, as a lender of last resort, as determiner of reserve requirements, and as monopoly supplier of currency are both unnecessary and harmful. Competition is beneficial in the production of regular commodities, and it should be in money goods as well.

The opponents argue that money is special, a natural monopoly as such, due to informational asymmetries and uncertain liquidity requirements. Without the restraint of regulation, as a natural result of profit or utility maximization issuers would not act in a way that benefits the society as a whole. Hence the need for government regulations such as deposit insurance and reserve requirements.

reminting (and revenue), gold coins retained 85% of their metal content.[2] Given the success with silver and Henry VIII's need for funds, why was he unable to extract more seigniorage by debasing gold coins?

Free Banking in the U.S. The 19th century U.S. banking system has been at the center of lengthy discussions about the need (or lack of need) for government to control currency and banking. In many states during this period it was extremely easy to start a bank and issue bank notes. There are examples of states that had numerous problems with failed banks, fraudulent banks, and bank runs that provide ammunition to arguments that government needs to tightly control private financial institutions. However there are also many states that seemed to have few problem banks. This of course is used as ammunition on the other side of the argument. Is there some reason that the degree of bank problems varied so much from place to place, rather than providing consistent evidence as to the general advantage or disadvantage to free banking?

Information, Incentives, Effectiveness. Before proceeding to the main discussion, we characterize some key underlying issues. I characterize the ingredients of an interesting social force as being information, incentives, and effectiveness. That is members of the population must have information, direct or indirect, to act upon. They must also have reason to adapt their behavior. That change in behavior must be effective, for example change the incentives for currency debasement by a monetary authority or enforce a new trading convention.

[2]See Chown (1994)

Ostroy (1973) have previously identified the essential role of money is to perform as a record-keeping device. This work emphasizes that holding money effectively serves as a social record that the holder is owed value. Thus the reason that people are willing to use an intermediate good in the process of trade, whether gold or paper, is that they expect they will be able to recover the value they have given up acquiring the intermediate good when they later trade it away.[3]

The informational aspect we highlight here is one which may be necessary to support the ability of money to serve its role as a good record keeping device. A monetary system conveys information about the activities of people in the economy. This could be directly, observing who is using money, or indirectly, through relative prices and the general price level. There are different useful types of information people might be able to learn through prices: the manner in which they allocate their resources to consumption and production activities, the usefulness of money in different segments of the economy, or as we will be principally concerned with in this paper, how and whether they should value money into the future.

People's current behaviors, such as trading, saving, and producing, depend significantly on what future changes in the monetary regime or the value of money they expect. We characterize the aggregate behavior of people as a social force.[4] Examples, which we discuss further below, include *bank runs, socially beneficial monetary con-*

[3]Ostroy (1973) shows that a competitive equilibrium may be impossible to implement with bilateral trades when agents demand *quid pro quo*. When a commonly acceptable record-keeping commodity is added, the allocation can be implemented through a series of single coincidence trades where the record-keeping good is on one side of every trade.

[4]We could call this an "invisible hand," but that term seems too narrowly associated with market phenomena.

ventions, and *flights from currency.* The crucial issue for effectiveness is whether the presence or anticipation of these social forces leads to (discourages) socially beneficial (socially inefficient) change. Thus, for example, the establishment and functioning of an exchange regime depends not just on the confidence that the population has in their ability to anticipate what the value of money acquired today will be when they go to buy goods in the future, but also the extent to which the information available to individuals and their response to it is sufficient to comprise effective community enforcement of this beneficial institution. We will also proceed to interpret in this manner failures and successes of attempts to collect seigniorage through debasements and fiat currency, and money issuance by unregulated banks.

1.2 Money Without Government or Banks

The benefits of exchange derive primarily from voluntary trade between individuals. Further even non-individual entities, such as firms, may trade in much the same manner. This provides ample justification for the recent approach of models of money as a medium of exchange, which abstract from coordinating institutions such as government and banks, despite a presence that seems ubiquitous and inevitable today. Examining these benefits, and other interesting properties of exchange that derive from the existence of monetary exchange, can be done without addressing the problems or benefits that might arise from such institutions.

In this essay we are not so much concerned with properties of monetary exchange

regimes as with the possibilities and problems for the existence of monetary exchange. Although this might seem to be an additional reason to require the presence of banks and government in the model, in fact, in their absence we will have something useful to say.

The Matching Model of Trade Frictions. By far the most common current paradigm for models of money as medium of exchange are related to the work of Kiyotaki and Wright (1989).[5] The environment introduced there emphasizes the trading frictions that result from the *double coincidence of wants problem.* This means that it is hard to find someone who both can produce the commodity you need and wants to consume the commodity you produce. In most versions of the model, an extreme case is used where there is no possibility of double coincidence of wants. The only non-autarkic possibility entails some form of indirect trade. Specifically, agents produce goods that are different from the ones they would like to consume. They then meet, often randomly, and attempt to trade, and after some sequence of trades acquire the goods they want to consume. One question that can be addressed in this environment, for example, is whether there are benefits to the use of a single common medium of exchange (monetary equilibrium) that are not present for other patterns of intermediated exchange. A commonly-used intermediate good reduces the number of types of trades that take place, these single coincidence trades are easier to find, thus money reduces the difficulty (time cost) of acquiring the final consumption good.

[5] Earlier pioneering work was done by Jones (1976) and Oh (1989)

1.2.1 Social Innovation of a Commodity Money

In these medium of exchange models, monetary equilibria exist as do non-monetary equilibria. Nash equilibrium, the relevant equilibrium concept here, means that no agent would benefit by unilaterally changing his trading strategy. And in the model there are no organized markets, no firms, and no government to coordinate the selection of one outcome over the other. Thus there is no mechanism to choose among multiple Nash equilibria, or dynamic to move between them. In particular, in these models there is no escape from a non-monetary equilibrium.

If, at the beginning of time, there was only barter exchange (a non-monetary equilibrium), how in this daunting environment could a monetary equilibrium arise? Clearly if there is to be an innovation it has to come from individuals. Chapter two of these essays addresses this issue. An environment is used where there is both the possibility for double coincidence trade and monetary trade.

Individual innovation, social innovation. It is shown that all that is needed to eventually bring about a "social innovation" is the inclusion of a small amount of noise in people's behavior. One one hand we do not want to take the abstraction from coordinating institutions too far; on the other hand, finding that only minor perturbations are required to reach the monetary equilibrium would indicate that whatever the actual source of the innovation mechanism – individual, entrepreneurial, or political – monetary exchange has robustness properties such that it will eventually be introduced and be far more successful than barter exchange.

An important issue discussed in chapter two is the length of time for a new trading convention to emerge. Even where one trading convention is more efficient and in a sense more stable, its establishment starting from another trading convention may take a very long time. This is especially true when this transition must be begun by an abrupt change. We establish there that when the change in trading convention can arise by way of intermediate small steps it can occur much more quickly.

Does this give any insight into the Persian experiment? The King tried to enact an abrupt change. To force all traders to use paper instead of their usual barter. An individual trader had no experience with the King's handling of a paper currency – very little certainty of what its value would be later – but could easily coordinate with his acquaintances to trade elsewhere in the usual way. That some traders would refuse to show up and use paper money meant that the trade was even less attractive to those who were more adventurous. It collapsed.

The historical examples of paper currency in China and in modern society have in common that the currency was redeemable for a long period. People were well established in using it to trade. When convertibility was suspended, temporarily or permanently, no change in trading activity means using fiat currency. The previous convention is gone, changed from above. To reject it would take coordination to establish an alternative or resorting to barter. Given the greater ease of monetary trade the transition is much more likely to succeed.

1.2.2 Social Enforcement

The ease of innovation in the above case results from the overwhelming robustness of monetary exchange as well as the size of trading behavior changes. What is the source of the robustness that gives such impetus to monetary exchange? One element is the big efficiency gain provided by monetary exchange (especially when compared to the only alternative in chapter two, direct barter). At the same time, an indispensable element in the robustness is the fact that this research has considered a situation where the quantity of the potential money goods is exogenously fixed. This type of robustness result would continue to hold when the money good varies so long as the variability is limited enough – either by some exogenous process or the inability of agents to produce it "too fast," perhaps due to the difficulty in mining gold or in writing out the notes.

Without limitation, say if individuals could freely issue pieces of paper, i.e. fiat currency, what would happen?[6] An agent who met another with a good he wanted, but without anything in return would be able to pay by issuing money. This would be accepted so long as it was easier to trade away the money than the original commodity. But as the money never disappears from the economy once issued, the quantity in circulation would continue to grow until it was no longer easier to trade than the original commodity. Money becomes an increasingly bad keeper of record.

The problem is the lack of restraint. There is no record of the issuer's behavior

[6]This situation is explicitly addressed in Ritter (1995).

and thus there can be no effective community enforcement of good behavior. So long as they did not have any external restraint, they would keep issuing currency until the value in exchange was the same as their cost of production. Thus if the currency was on paper, and the notes could be produced faster than society grew, the society would end up with a money eventually worth only the paper it was printed on, ie. nothing.

Fixing an exogenous supply of the commodities takes the most direct route to solving the problem: it is assumed away. There is no opportunity for surprising expansions of the supply of money and loss of value by those who hold it. This supports the historical use of commodities, such as gold, that were costly to mine and so had a fairly regular supply

Still, it is not transparent that relaxing this assumption so that agents could potentially over-issue must necessarily lead to the failure of monetary exchange. In particular if the anonymity assumption were relaxed so that agents who over-issued faced consequences, could private issue be enforced by the information acquired through trade? In essence, each agent would be a bank.

1.3 Government

We are not interested in examining government control of money, per se, but government's ability to use money, fiat or commodity, for revenue extraction. Given the high social value of a monetary system and its potential as a source of revenue (as discussed below) motives for the establishment of monetary systems, for either social welfare concerns and for sovereign enrichment, are not hard to find. In fact whenever non-governmental exchange media have become successful, governments have usually been quick to claim exclusive rights to issue or regulate money such money.[7] What we are interested in is how population behavior can affect the ability and incentives to collect seigniorage.

1.3.1 Social Enforcement in Commodity Money Systems

Debasement. Minting was the exclusive right of the sovereign, who could extract seigniorage by taking in metal (either in plate or in old coins) and returning newly minted coins with metal content was less than that taken in. The amount of revenue generated depends on the rate of seigniorage (the difference between metal taken in and metal paid out) and the quantity of coining.[8]

Why would people have coins reminted if there was a loss of metal to seigniorage? Coins carry a face value so there two ways to value them: by face value (tale), and by intrinsic content (specie). To encourage people to bring in old money, reminting that resulted in reductions in the content of silver or gold was often accompanied by a premium in face value. This means that a person who brought to the mint coins with total face value of one pound would leave with less metal but more than one pound in nominal value of the coins. If the new money could be spent at or near par

[7]See Davies (1994) and Tullock (1957) for some examples.

[8]For a debasement, the reduction in metal content of newly minted coins, to be successful, it must be accompanied by a lot of minting activity.

with the old, this would encourage people to get their old money recoined.

Does it work? In order to work, coins must be more valuable than their intrinsic content. Otherwise people would not bring in coins to be reminted and they wouldn't bring in plate if they were to receive less back than the value of the increased convenience of coin. We need to consider the source of the gap between the value of a coin and the value of its content, how difficult it is to maintain a gap, and what happens as it shrinks.

Coins are convenient in exchange because they have identifying marks that are easily recognized, while uncoined metal is difficult to identify by a non-specialist. The premium for coin over the commodity it contains can occur when coins are the medium of exchange and are costly to counterfeit because they have a use that the commodity does not (exchange) and thus their price can be above that of the constituent commodities.[9]

Quantity theory arguments (assuming no other changes in the economy) make clear that the debased currency can not maintain its value unless the nominal quantity of coin in circulation is held constant. Thus the aggregate plate must be reduced and the government must sell its seigniorage as plate. But when there is a gap between the value of coins and their content-equivalent quantity of plate, then the plate the government collected as seigniorage would be much more valuable if coined before

[9]And though it can be above, it won't be below. The conversion from plate to coin is controlled by the government, but the other direction, from coin to plate, can be done fairly easily. If the plate content is more valuable then the exchange value of the coin, coins quickly disappear from circulation.

being spent. If the government yields to this temptation, as it seems that Henry VIII did, the quantity of metal in circulation stays roughly constant rather than the nominal quantity, which increases. Inflation increases the price of all goods, in particular gold and silver, and the gap is reduced.

Does this lead to any difficulty for government to collect seigniorage? Historically the use of a currency seems very robust even in the face of severe inflations, as Germany in the inter-war years and recent Latin American experiences show. But people do respond to inflations and debasements.

Flight from Currency As people learn the behavior of the government, they begin to anticipate that future prices will rise and that coins will continue to lose their metal content. They recognize the loss of metal faster and prices adjust quickly. People go for reminting because they directly benefit from the nominal gain they receive.[10] However when price adjustments become more rapid, individuals must spend quickly to avoid losing all the gain to inflation.

The issue is how do people respond. One response is to change how coins are treated - to price them by their content. As the gap between the exchange value of coins and content value shrinks this becomes more viable. But is costly for traders, especially at a small scale, to keep up with the content-value of all the different coins. Another response is to adapt non-spot transactions to account for future changes in prices. Thus contracts may recognize an expected rate of inflation or specify that gold

[10]This rapid disappearance of higher quality coins is part of the cause of Gresham's Law. The other part also works to drive out of circulation more valuable old coins, becuase their better plate content makes them a better store of value, out of circulation.

(or equivalent) be used for payments.[11] To the extent that these responses occur, the incentive to remint is reduced.

Ability to debase: Gold vs. Silver. Both responses are more difficult for small transactions, so it is here that the burden is going to be heaviest. Thus there is more evidence of gold coins being exchanged at content value then silver coins during the Great Debasement. For small purchases it was difficult to avoid using silver coins. Gold coin was of a larger denomination than silver coin, and would have often been held as a store of value rather then used in everyday exchange. Even though still inconvenient, given their higher value it was relatively less costly to ascertain its intrinsic content then an equivalent pile of silver coins. Furthermore gold was more often used in international transactions where traders would refuse the devalued silver coins at face value.

Apparently the general public had little recourse but to rely on the face value of silver coins, however devalued, in general exchange. The parties to larger transactions that were carried out using gold or by contract were more able to avoid the costs and risks of debasements.[12] It is difficult to come up with reliable figures, but even though the debasement of gold was much less (retaining 83% of its content versus 25% for silver) and seigniorage rate was much lower (the maximum for gold was 15% versus 55% for silver at the same time), the quantity of gold minted rapidly fell off even as

[11]There is some discussion of this in Rolnick, Velde, and Weber (1997) and Chown (1994) pp. 49-51. A modern example of this is a job the author is currently preparing to take in a country with a history of currency fluctuations. Although the salary is paid in local currency, the contract does not specify the amount. Instead it specifies that the amount of the equivalent U.S. dollars.

[12]Seignorage functioned here as a very regressive tax.

silver continued to be minted in large quantity.[13]

Gold was used largely as a store of value and trades carried out with gold were large enough transactions that trading by gold content was not too costly. Silver coins, as the medium of exchange, provided higher rents to traders. Debasements of silver coin were able to extract rents from the more inelastic demand for silver minted coins. This response was effective in the sense that it eliminated the profitability of debasing gold.

1.3.2 Social Enforcement in Fiat Money Systems

Commodity backed paper currency systems are subject to much the same debasement considerations as those with coinage. However even governments that were more restrained than Henry VIII have faced times, such as during wars, when they could not meet convertibility demands. Experiments with convertibility suspension led to the consideration of the possibility of permanent suspension, ie. fiat currency.

Governments would tend towards over-issue of currency for seigniorage. What does it take for credible government currency? We will not discuss political means of control. But there is research that should be mentioned that discusses how a self-interested government can establish credibility.

Ritter (1995) shows that self-interest can be a sufficient motive for a government to restrain itself from issuing too much currency. Members of government also hold currency and are also affected as currency increases. A sufficiently large government

[13]These figures are cited in Chown (1994), see especially table 5.4.

would maintain a currency level that balanced its gain from issuing currency (seigniorage) with its loss from the increased difficulty of trade in the presence of too much money.

Knowledge of the level of the money supply may permit self-interested rational government from over-issuing if it is patient and large enough. The size requirement and an (undiscussed) relationship to number of commodities don't make a lot of sense to me. While not necessarily a problem, it is because the paper confounds medium of exchange motivations with seigniorage motivations in government agents that this result is achieved.

1.4 Competitive Banking and Currency

One aspect of the discussions in the free banking literature and one that has been active recently is whether an unregulated banking system has the ability to function properly.[14] We discuss two strands of literature, the first, based on the work of Diamond and Dybvig (1983), focuses on liquidity. This work has generally focused on the ability of a depository institution to successfully balance the conflicting needs of meeting uncertain liquidity demands and directing resources to their most productive (less liquid) uses. The second looks directly at currency issue in a search environment similar to those discussed above.

[14]For two contrasting views in this literature see Goodhart (1988) and Dowd (1993)

The Diamond-Dybvig Model of Bank Runs from Early Withdrawals. This environment incorporates uncertain timing of demand for real resources. Efficient use of resources typically requires that the resource be committed for some period of time. Agents have an endowment and an uncertain (and unknown) lifespan of either two or three periods. Agents wants to consume a lot in their final period of life but they do not find out their lifespan until the second period. There is a productive storage technology that takes a two period commitment to give a high return or can be liquidated early for a low return. Because they are risk-averse, an insurance contract that increased second period consumption if an agent turned out to be short-lived would, ex-ante, make all agents better off. Because there is a known fraction of agents who will die young, there is an optimal contract that would accomplish this. Unfortunately the information about lifespan is private information, so a conventional insurance contract is not incentive compatible – each agent would find it in his interest to claim that he is short-lived and get the extra consumption.

Diamond-Dybvig proposes an institution that implements this contract. That is, agents deposit their endowment with an intermediary who gives one return to agents who come to withdraw in period two and another return to agents who come to withdraw in period three. The bank pays deposits sequentially as agents arrive as long as there are funds and the next period the surviving agents split the remaining proceeds. The intermediary gives the appropriate return to early-withdrawing agents so that agents will have no incentive to misreport their lifespan. There is a good

equilibria where agents withdraw in the appropriate period, however there is another equilibria where each agent believes that all other agents will try to withdraw early, so to get any consumption he must try to withdraw early also. This is characterized as a bank run.

1.4.1 Social Enforcement in a Backed Money System

Recently Williamson (1992) has argued against free banking and for the necessary role of government in regulating money. In an over-lapping generations model, as developed by Diamond and Dybvig (1983), agents choose whether to save by holding banks liabilities or fiat currency. Banks are short-lived agents that issue claims backed by an investment. This investment can be either in expensive good assets or bad assets that are less costly. When this information is private, the lemons problem results in an inefficient outcome where the only circulating assets are backed by bad assets. When the issuance of bank liabilities is prohibited (by government) all that remains are autarky and an equilibrium where money is held.

In a sense Williamson has stacked the deck against bank money. The lemons problem may apply to bank money, but bank notes circulate with the name of their bank and even in the U.S. free banking period there were periodicals giving discounts on the liabilities of the various banks. Arguments in support of free-banking appeal to the benefits of competition. Although Williamson's banks are competitive in pricing their obligations, the limitation to one-period lives gives no opportunity to compete on the basis of reputation. This features centrally in the arguments put forward in Klein (1974).

In Klein firms issue differentiated currency. Thus, as in the 19th century, bills circulate that have firms' names on them. Klein argues that banks' incentive to maintain their brand-name capital will deter them from over issuing currency. People have a demand for the services from real money balances. He considers two situations; first assuming that each bank's future behavior is commonly known, and second assuming that there is a "consumer confidence" variable that is related to the predictability of the banks money supply.

If the the quantities of money issued by banks are perfectly anticipated, then the prices of the currency will reflect the entire history and future actions by banks (this is similarly true for the earlier discussion of Henry VIII). So it doesn't matter what those actions are specifically as long as they are anticipated. A problem with this reasoning is that it is only reasonable to assume a common knowledge of the banks' behavior if there is no incentive for them to deviate. The mechanism by which deviation would be punished is not discussed. The zero profits condition effectively limits the capacity for punishment for deviations. The second case that Klein discusses, where predictability is known fares no better in this regard. What is the punishment that removes the incentive for a bank to suddenly repudiate its obligations, as wildcat banks were famous for doing. Apparently the more competitive the market the less effective the enforcement mechanism.

1.4.2 Social Enforcement with Competitive Fiat Money

Could a fiat currency be supplied by competing banks? Recall above that Ritter (1995) argued the currency issuer must be large relative to the size of the economy so that he would have sufficient credibility to avoid over issuing. This suggests that individual agents could not be issuers, however the key is that the issuer must benefit from being a properly behaving issuer and there must be a mechanism by which he can be punished for behaving improperly.

Recently Cavalcanti and Wallace (1997) have developed a random matching model in which there are two classes of agents, those that trade anonymously and those that have their history of trade known. The public traders (who we can call "banks") are also given the ability to issue distinguishable notes. The number of banker-agents is fixed and they profit from that status. Because their behavior is know and they are not anonymous they can be punished for failing to abide by an equilibrium. Thus the authors are able to find stationary, incentive compatible equilibria where banks issue and redeem their own notes.

Again the key element is the information about behavior and the ability to punish of deviating banks (those that refuse to redeem bank liabilities). The need for bank profits (and thus the exogenous restriction on the number of banks) could be taken to be an argument for regulation in the economy. In a truly free free-banking regime banks would have relatively little operating profits (because of the ease of entry by competitors) and thus the loss of these would be a relatively small deterence from repudiating their debt.

In fact the actual 'free banking' systems that existed in many states in the U.S. in the 19th century, were not completely free and contained a number of checks that helped enforce good behavior. In a number of states liability by shareholders were liable at default for an amount equal to their initial investment (double liability). Additionally there were reserve requirments in the form of maintaining state or federal bonds on deposit with the state license.[15]

The majority of problems in states with 'free banking,' discussed by Dowd, occurred due to state default on the bonds that were heavily used as reserves. This was the case in Illinois, Wisconsin, and Indiana. It is unclear how much of this risk was knowingly accepted and how much was perhaps part of a lemons phenomena.

Dowd traces problems with 'free banking' in Michigan to the decision of the state to lift the specie convertibility requirement of bank notes. Redemption operated as an important check on bank behavior. This removed much of the risk to bankers of over-issuing currency. In particular, information about issue behavior was reduced as there were no redemptions to show their lack of currency reserves and they could evade detection for a longer time.

[15] See Dowd (1993) chapter eight for discussion and further references.

Chapter 2

The Emergence of Money as a Medium of Exchange

2.1 Introduction

What is required for an economy to switch from barter to monetary exchange? How stable is the institution of monetary exchange? Since early expositions on the difficulty of finding the double coincidence of wants necessary for direct barter, it has been clear that a monetary system of exchange (where some type of good is generally acceptable as a medium of exchange) is of great value to society and the individuals that compose it. People give up valuable goods in exchange for something that they typically cannot consume or use in productive activities. Their intent is to trade at a later date this intermediate item for something they do want. They anticipate that two indirect trades to get from an initial to a final commodity bundle repudiating their debt.

In fact the actual 'free banking' systems that existed in many states in the U.S. in the 19th century, were not completely free and contained a number of checks that helped enforce good behavior. In a number of states liability by shareholders were liable at default for an amount equal to their initial investment (double liability). Additionally there were reserve requirments in the form of maintaining state or federal bonds on deposit with the state license.[15]

The majority of problems in states with 'free banking,' discussed by Dowd, occurred due to state default on the bonds that were heavily used as reserves. This was the case in Illinois, Wisconsin, and Indiana. It is unclear how much of this risk was knowingly accepted and how much was perhaps part of a lemons phenomena.

Dowd traces problems with 'free banking' in Michigan to the decision of the state to lift the specie convertibility requirement of bank notes. Redemption operated as an important check on bank behavior. This removed much of the risk to bankers of over-issuing currency. In particular, information about issue behavior was reduced as there were no redemptions to show their lack of currency reserves and they could evade detection for a longer time.

[15] See Dowd (1993) chapter eight for discussion and further references.

Chapter 2

The Emergence of Money as a Medium of Exchange

2.1 Introduction

What is required for an economy to switch from barter to monetary exchange? How stable is the institution of monetary exchange? Since early expositions on the difficulty of finding the double coincidence of wants necessary for direct barter, it has been clear that a monetary system of exchange (where some type of good is generally acceptable as a medium of exchange) is of great value to society and the individuals that compose it. People give up valuable goods in exchange for something that they typically cannot consume or use in productive activities. Their intent is to trade at a later date this intermediate item for something they do want. They anticipate that two indirect trades to get from an initial to a final commodity bundle are quicker than one direct trade. In a world of diverse trading goals, where the time available for trading is scarce, it is more efficient (trades occur more quickly) when some commodity (or commodities) serve to expand the number of potential trading partners by reducing the types of trades that are made. However, the value an agent attaches to the intermediate good, and thus his willingness to accept it in exchange, depends entirely on the willingness of other agents to accept it. Given this strategic uncertainty, it is not transparent that a medium of exchange could come into use in the absence of government or some other explicit coordination device.[1]

The emphasis here is on the social coordination aspect of exchange. Models incorporating this were introduced by Jones (1976) and later Kiyotaki and Wright (1989). In these papers agents are endowed with or produce some commodity but prefer to consume a different one. Pairwise meetings occur at random and paired agents choose what types of trades to make (ie. which commodities they would accept in return for their current commodity). Different compositions of trading strategies have different efficiency implications. Especially interesting are equilibria that resemble real world patterns of exchange. The structure of these models emphasizes the relative efficiency of intermediated trade, especially in Kiyotaki and Wright, where no double coincidence opportunities exist.

We focus on the question of what conditions are necessary for monetary exchange

[1]This is the question posed by Menger (1892) of "whether the transition from barter to monetary exchange could be expected to take place spontaneously under the pressure of market forces whenever it promised welfare gains or whether it required an 'invention,' combined with persuasion, convention, or compulsion," Niehans (1978), pp. 2-3.

to be the unique stable state of the economy (in a manner made precise below). Additionally, it will be eventually be adopted over direct barter regardless initial conditions. In this paper we construct a simple model that contains basic features that make social coordination desirable, as in the earlier models. We show that if an economy is diverse enough in its collection of commodities, regardless of initial conditions, it will eventually develop a social convention that is characterized by a monetary system of exchange. This convention will emerge in finite time from any initial state and can be expected to be maintained for a long time (relative to how long it was expected to take to get there).

Kiyotaki and Wright look for steady state equilibria by using stationarity and individual rationality conditions. Many equilibria of this type exist, autarkic equilibria and different patterns of indirect exchange (some of which are monetary exchange patterns) are examples. However, because of their reliance on steady state conditions, they cannot analyze the model outside of steady state equilibria. In particular they cannot eliminate equilibria based on stability. Thus, with no mechanism in their model to select among equilibria, they are not able to make definite statements on the conditions under which a particular pattern of trade would be observed.

Renero (1995a) has examined equilibrium dynamics in a specific version of the Kiyotaki and Wright model. He shows that not only do different initial conditions lead to convergence to various steady states, but also that some steady states that appear economically sensible are unstable while there are less sensible ones that are stable.

Earlier work by Oh (1989), building on the research by Jones, also had the property that which steady state the economy converged to depended on initial conditions. Thus these models lack an endogenous mechanism to select among multiple stable equilibria.

Our aim is to begin a more comprehensive analysis of social patterns of trade. To do this we use the recent literature on stochastic evolutionary games (Kandori, Mailath, and Rob (1993) and Foster and Young (1990)). This research introduces noise into agents' choice of strategy. While it is reasonable that people make some mistakes, it is not acceptable to assume that people make frequent mistakes or often hold their behavior fixed when they really should correct it - especially when this irrationality is costly. Thus the literature focuses on the effect of the inclusion of arbitrarily small trembles. When combined with a way agents use information about history to form strategies, dynamics can be characterized that are ergodic, eg. the system is not dependent on initial condition for model behavior in the distant future (even though they might be very important in the short and medium term). Thus, this perturbation to the model induces a distribution over states so that some equilibria can be characterized as being observed so infrequently as to be irrelevant and others, or perhaps just one, as having stability properties so that they are most likely to be observed in the long-run. Additionally the stochastic evolutionary approach allows predictions about how long before a particular equilibrium is achieved from initial conditions.

In this paper, as in the earlier models, we construct a simple model that contains basic features that make social coordination desirable. We focus on the question of what conditions are necessary for monetary exchange to be adopted over direct barter from any initial social pattern of exchange. This leads to some differences between our model and the Kiyotaki and Wright economy. Unlike their model, we allow direct barter and we allow only one commodity to be used as an intermediate good. We assume an exchange economy with finitely many commodities, including a "token" commodity that, although unconsumable, has the capability to be used as a medium of exchange. Although exchange is necessary to consume, indirect exchange is not. Agents choose whether to look for direct exchanges or to accept the token commodity as a medium of exchange. Using tokens is risky because although there are always opportunities for direct barter, an agent who accepts the token commodity may find it impossible to locate another agent with whom to trade if other agents do not also use the tokens. Many patterns are equilibria, including both the monetary exchange pattern (where all agents accept tokens) and the direct barter pattern where all agents seek out traders with a double coincidence of wants.

We then examine what the addition of a "best-response" learning rule and a small amount of noise means for the evolution of the economy. This enables us to show that if there are enough commodities in the economy, monetary exchange will be adopted. We also show that the length of time required for this convention to emerge is decreasing in the number of commodities. We also show how the quality of information (degree of detail) that agents have about the population can be crucial in speeding up the transition. This speedup is a somewhat asymmetric implication of the model, information about changes in the behavior of producers (groups of agents who are endowed with the same commodity) can result in large changes in population strategies while information about the behavior of similarly grouped consumers does not. Where the information lacks the detail about producers, convergence is much slower.

2.2 The Model

In this section we construct a finite economy composed of agents with diverse trading goals and limited opportunities for exchange. In order to get a commodity they can consume, agents must make exchanges (which are based on a trading strategy they choose).

Assumption 1 *There are $l+1$ types of commodities, all indivisible.*

Assumption 2 *Each agent has a type $\theta = (\theta^e, \theta^c) \in \{1, \ldots, l\} \times \{1, \ldots, l\}$, $\theta^e \neq \theta^c$, where θ^e is the type good he is endowed with and θ^c is the type of good he wants to consume.*[2] *There are n agents of each type and $N = n \cdot l(l-1)$ in total.*

Assumption 3 *The $l+1$ commodity, which we call the* token *commodity, is not consumed or produced by anyone. It is also special because it is storeable. An agent can hold one unit of either his endowment commodity or the token commodity without inventory costs. He cannot hold more that one unit and nor can he hold any other commodity.*

[2] *Although the assumption that agents have a single good and want a single good appears restrictive, we could consider agents who make repeated trips to the market with different goods until they have a "bundle" that they desire.*

Assumption 4 *Agents make only anonymous bilateral voluntary exchanges. These exchanges are restricted to direct barter (double coincidence of wants) or indirect barter with a token as the intermediating commodity.*

Assumption 5 *Agents receive utility from consuming their consumption-type commodity and none from any other commodity. They discount consumption that occurs later at rate β.*

2.2.1 The Game

Agents choose a strategy s from the set $\{A, D\}$ which specifies the trades they are willing to make conditional on the good they hold when opportunities arise.[3] Those who adopt strategy D will only trade their endowment directly for their consumption good (direct barter). Those who adopt strategy A may also make direct barter trades, but will additionally accept tokens and then try to trade the tokens for their consumption good (indirect barter).

Based on an agent's beliefs about the environment over the relevant horizon he chooses the strategy that maximizes his payoff. Having chosen his strategy; with probability m the agent is given the opportunity to accept a unit of the token com-

[3] For the present we consider only pure strategies.

modity in place of his endowment. If he adopted strategy D he retains his endowment. if he adopted strategy A he accepts the token, and his endowment is lost.

The population, divided into types, is now subdivided by the strategy of those agents and by the holding of the agents. As we see in figure 2.2.1 (where we have for illustration five commodities and consider agents of type $(1,5)$), the agent of type θ could have adopted: strategy D (and hold his endowment), strategy A and hold his endowment, or strategy A and hold the token commodity. The number of these agents is denoted by d^θ, a^θ, and m^θ (respectively). Define c^i as the set of agents who consume good i (those in column i) and e^j to be those who are endowed with commodity j (those in row j). Referring again to the economy in figure 2.2.1 a type $\theta = (1,5)$ can potentially trade only with agents in one or both of the sets e^5 and c^1.

Each agent in the population is randomly paired with another agent. They trade (or not) based upon the strategies they adopted. Agents who successfully trade are re-endowed immediately and return to trade again with the same strategy. The random pairings continue, but for each agent the end of trade arrives randomly. At that time the agent is replaced and his inventory is lost to the economy. For an individual agent the effect of this, combined with other sources of impatience, is a per-period discount factor of β on future consumption.

An agent will always want to make a trade that gives him his consumption good. If a trade would give him a token, he would want to decide based on whether or not he could more easily acquire his consumption good in the future. Thus the agent's

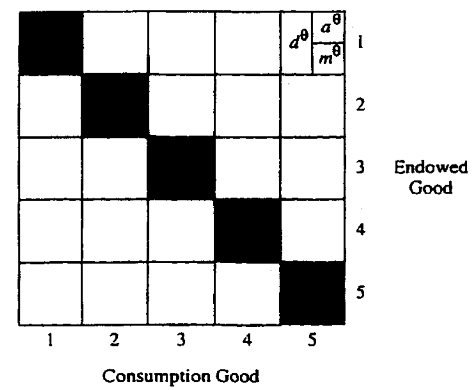

Figure 2.1: Agents in Type Space (5 commodity illustration)

entire concern is with present and future trade probabilities. The probability of a θ agent who holds his endowment good acquiring his consumption good at time t is

$$q_t^\theta(c|e) = \frac{1}{N-1}\left[a_t^{\theta'} + d_t^{\theta'}\right],$$

where θ' is the type that has a double coincidence of wants with θ. For an agent who holds his endowment and uses strategy A, the probability of trading for a token is

$$q_t^\theta(m|e) = \frac{1}{N-1}\sum_{\hat\theta \in c^\theta} m_t^{\hat\theta}.$$

token good, the probability of acquiring his consumption good at time t is

$$q_t^\theta(c|m) = \frac{1}{N-1}\sum_{\hat\theta \in c^\theta} a_t^{\hat\theta}.$$

The trade decision is more complicated when these probabilities are changing over time, in particular if sometimes tokens are better and sometimes the endowment is better. We can characterize the decision simply if the following condition is satisfied.

Lemma 2.1 *An agent of type θ prefers strategy A at time s if his belief is that*

$$q_t^\theta(c|m) \geq q_t^\theta(c|e),$$

for all $t \geq s$. Equivalently, if

$$\sum_{\substack{\hat\theta \in c^\theta \\ \hat\theta \neq \theta'}} a_t^{\hat\theta} \geq d_t^{\theta'}.$$

An agent prefers strategy D if the inequality is reversed for all t.

Proof: See appendix.

Even under the Nash equilibrium requirement that these beliefs are correct, we can see that there are still at least two outcomes that might be observed in this economy.

Corollary 2.2 *In any exchange economy of this type, there are at least the following Nash Equilibria:*

1. *Pure direct barter in which all agents use strategy D.*
2. *Monetary exchange in which agents use strategy A.*

Proof: This follows trivially from Lemma 2.1.

Typically there will be other Nash equilibria as well. Specifically there may be the following types:

1. All agents in both types of some matched pair, θ and θ', use strategy A while all other types use strategy D.
2. All agents of types in θ_i^e and θ_i^c (all those who either are endowed with or consume i) use strategy A, all others use strategy D.

Suppose that parameters were chosen so that if none of an agent's double coincidence types were using tokens, it then would take k other agents who produce her consumption commodity to use tokens to get her to switch to using tokens also. Thus if all of one of these types (given by a production, consumption good pair) were to use tokens and there is less than k of that type it wouldn't be enough for her to switch.

It would be enough for that type's direct-trade type agents to switch (as discussed above). This may be one Nash equilibria. There may be others, for example there may be one "sector" (both producers and consumers of a particular good) which is entirely strategy A and some pairs of types (see item 2 in the corollary) outside of the sector that also use A in a Nash Equilibria.

We will say more below about the problem of multiple equilibria and the method we use to address the issue in section 3. First we consider the two equilibria in Corollary 2.2 in more detail. This serves the dual purpose of characterizing the value of monetary exchange in different economies and emphasizing why a model that is able select among outcomes with such widely variant welfare properties is important.

2.2.2 Efficiency of Monetary Exchange vs. Direct Barter

In this section we characterize the relative efficiency of the two Nash equilibria that interest us most ("all A" and "all D"). In any situation such as both of these equilibria, where an equal fraction of each type plays strategy A, it is straightforward to show that the symmetry implies that trade probabilities are constant over rounds. Thus we get the following value functions for agent's in each equilibria using Bellman's equation (refer to the proof of lemma 2.1).

$$V(\text{All D}) = \frac{\beta}{(1-\beta)(l^2-l)+\beta}$$

$$V(\text{All A}) = \beta(1-m)\cdot\frac{\frac{1}{l-1}+\frac{\beta m}{(1-\beta)l+\beta(1-m)}}{l(1-\beta)+\beta\left(m+\frac{1-m}{l-1}\right)}$$

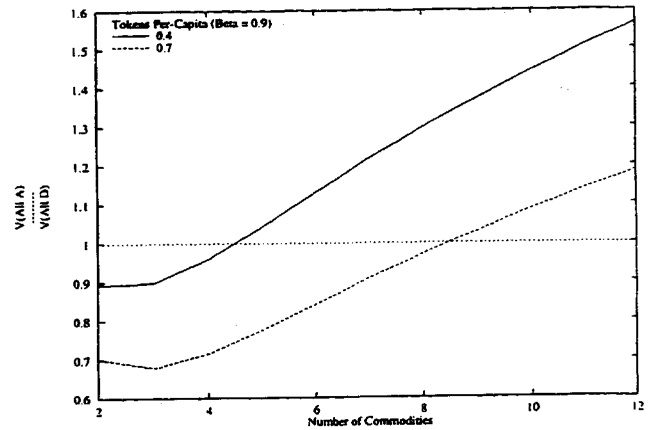

Figure 2.2: Relative Value of Equilibria vs. Number of Commodities

The comparison of these equilibria is not transparent (especially from the appearance of the second expression). We are mainly interested in the relative value of the two equilibria and with some algebra it is not too difficult to see that although both go to zero as l becomes large, the ratio converges to

$$\frac{V(\text{All A})}{V(\text{All D})} = (1-m)l.$$

Thus it becomes increasingly better to be in a monetary equilibria as the economy becomes more differentiated. Figure 2.2 illustrates this using the exact expression. It should be emphasized that this represents only relative trading efficiency and not gains to specialization.

Although a thorough analysis of the relative value of the equilibria is not the focus

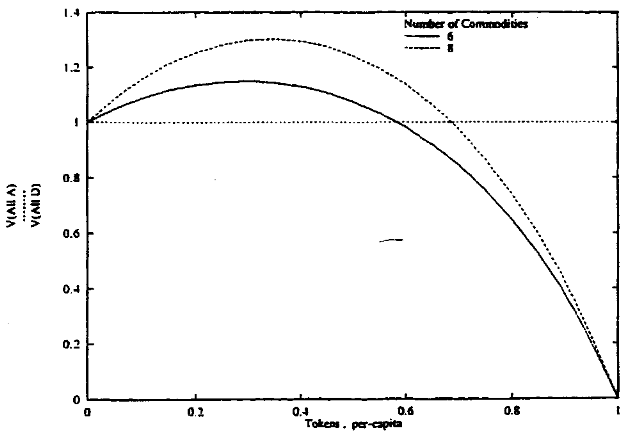

Figure 2.3: Relative Value of Equilibria vs. Quantity of Tokens

of this paper, for intuition we provide several graphs to illustrate relative efficiency as the parameters β and m vary.

The above approximate ratio for large l, makes it appear that tokens impose an efficiency cost on society. In fact, as Figure 2.3 illustrates, the effect of the per-capita token stock on efficiency is not so simple. With no tokens the values are equal (as all trades will be direct in either equilibria), larger per-capita token levels increase efficiency until an optimum is reached. However for every token floating around the economy, a unit of a consumable commodity had to be sacrificed. The inability to hold both tokens and endowment imposes a real cost on society that both emphasizes the gains (it is efficient to sacrifice real output, even a lot of it, to trade more quickly)

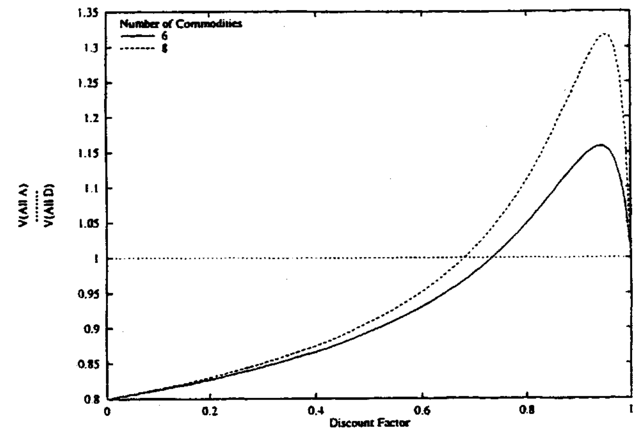

Figure 2.4: Relative Value of Equilibria vs. Discount Factor

from intermediated trade and understates these gains (unless it is for commodity money, few resources are really sacrificed in carrying or constructing a mediating commodity).

Figure 2.4 illustrates that although the determination of the efficient equilibria can be sensitive to the discount rate, the monetary equilibria still becomes strictly better as the number of commodities increase.

2.3 Dynamics

We have shown that the relative efficiency of equilibria may differ across economies. Our goal in this section is to see whether society can achieve the better equilibria.

We introduce a dynamic environment that will help to evaluate the viability of the equilibria in our model. Search models of monetary exchange have largely accepted multiple equilibria as a necessary result of their steady state analyses. While this permits efficiency comparisons between equilibria it doesn't help enough in evaluating the likely outcome in such a world. Questions about the stability of monetary exchange and the ability of monetary system to emerge without coordination are thus difficult to address with such an analysis.

Work on a Kiyotaki and Wright type of model by Renero (1995a) using initial conditions has highlighted the need for caution in accepting equilibria at face value. Furthermore, the existence of multiple equilibria in a model is something that has been viewed as a modeling weakness. This is because of two notions of what makes a good theory; it should be able to eliminate any counterintuitive equilibria and should contain clear predictive content. Models with multiple equilibria often fail on the first test and by definition fail the second.

The game theory literature wrestled with these issues through a long series of refinements. Several of these refinements are based on the idea that "good" equilibria should be stable in the face of small amounts of noise. Kandori, Mailath, and Rob (1993) and Foster and Young (1990) elaborated an explicit stochastic dynamic framework that takes seriously these stories of "mistakes" and evolutionary dynamics that emerged out of the refinement literature. The dynamic structure specifies what *information* agents have about the history of play and state variables and the

learning rule they use to determine their strategy.[4] A small amount of *noise* (due to unmodeled factors such as experimentation or mistakes) is then added. The presence of these two elements permits the economy to be outside of the equilibria and still have a defined dynamic. As the noise gets small the amount of time spent outside some states (called stochastically stable) becomes small. Not only does this allow the set of equilibria to be reduced, consideration can be given to the manner and frequency with which transitions occur over the long-run. Thus the analysis allows a detailed portrait of social dynamics to be drawn.

Information. We assume that agents are given information directly about the strategies in use.[5] We will analyze the model under two alternative assumptions about how much information is available to agents. That agents know "everything" (the distribution of strategies across types) is one assumption. The other is that agent only have access to an aggregate statistic – the fraction of the whole population using each strategy. Examining the difference the added information makes is one motivation for considering both cases, additionally the simplicity that results in the aggregate statistic case will simplify the analysis there considerably.

Learning Rule. When imposing a learning rule, a right or best method of inference is not self-evident, but two conditions that a reasonable inferences should

[4] One effect of the structure we have specified is that the future path inventory holdings is always the same when agents make their strategy choices. This reduces the complexity of the problem by eliminating money and commodity holdings as a state variable. The state then consists solely of the strategy profile of the population.

[5] From a perspective of realism it would be desirable to have agents infer about strategies from observable data such as trades. However it isn't clear that there would be any impact on the results and the cost to the clarity of the analysis would be high.

satisfy are: first, they usually should be correct, and second, avoidable mistakes shouldn't be either too costly or too persistent. A rule that will often satisfy these criteria, is easy to apply, and has been widely used in the literature, is the best-response dynamic.[6] Each agent makes a best response to his information about the strategies in use. Thus every static Nash equilibria (in which this knowledge is both complete and correct) is also potentially an equilibria in the dynamic (which we note is a Markov process).[7]

Finally in defining the intentional population dynamic, we recall that agents, either due to the difficulty of evaluating alternatives or otherwise only occasionally change their strategy (as they are replaced in the population). This gives additional support to the use of best-response learning (the slower the environment changes the "better" is the best-response). This process of replacement is given by the probability δ replacement of each agent. However the amount of "inertia" doesn't have any effect on stochastic stability, it only slows the dynamic. Thus although the results hold generally, we can simplify the analysis by ignoring periods where no one is replaced and further reduce notation by analyzing the situation where replacement is perfectly correlated (all agents are replaced at the same time).

The state is $z_t \in Z$, where Z contains all 2^N possible strategy profiles. It is convenient to define z_t as a 2^N-vector, consisting of all zeros except for a one in a position that indicates the state. The information structure $\Omega(Z)$ maps the 2^N states

[6] A different learning rule could be the fictitious play of Fudenberg and Levine (1993).

[7] Cycles are also potential phenomena, this would be troubling because agents don't have to be very smart to take advantage of such phenomena.

into K observable states (observe that $K = N + 1$ in the aggregate information case). Given the information a type θ agent has about the state, he makes a best response. The state that results from the combined best responses of the population is $b(\Omega(z_t)) \in Z$.

We can now define the intentional evolutionary correspondence, $P : Z \to Z$, so that $z_{t+1} = b(\Omega(z_t)) = z_t P$, where P is a $N \times N$ matrix of transition probabilities. When the best-response function is single valued this consists of zeros and ones.

Noise. We impose a small probability, ϵ of each agent making a "mistake" in selecting his strategy. This could be considered to be experimentation (though not optimally chosen). The resulting Markov process is given by $z_{t+1} = z_t P(\epsilon)$ where

$$P_{z_t, z_{t+1}}(\epsilon) = \epsilon^{c(z_t, z_{t+1})}(1-\epsilon)^{N-c(z_t, z_{t+1})},$$

and $c(z_t, z_{t+1})$ is the *transition cost*, the number of agents who have different strategies differently in state z_{t+1} than they would have held given their best-response, $b(\Omega(z_t))$.

The state transition matrix $P(\epsilon)$ has strictly positive elements and hence it has unique ergodic distribution.[8] Let μ^ϵ be the stationary state distribution, then the limit distribution is defined by $\mu^* = \lim_{\epsilon \to 0} \mu^\epsilon$ and *long-run equilibria* elements of μ^* which are greater than zero. In addition to this process on the "true" state we can define another on the "information" state. This is convenient to work with because it is of much lower dimension.

[8] Ergodicity doesn't require all positive elements. There could be zero elements as long as all states communicate (there is a positive probability of moving between them in finite number of steps). Our assumptions allow us to avoid having to work very hard to show ergodicity.

2.3.1 Case 1: Aggregate Information Only

Suppose that the only information available to agents is the number of agents that used strategy A in the previous period, thus there are $N+1$ possible states. Here the information has the same impact on each agent's decision. Thus there are only three possible equilibria: all agents use strategy A, all agents use strategy D, and agents use strategies such that all are indifferent.

Lemma 2.3 *For economies with non-trivial stocks of tokens, ie.* $0 < m < 1$, *and a discount factor* $0 < \beta < 1$, *the unique long run equilibria has all agents choosing strategy* A *if*

$$l > \frac{1}{1-m} + 2$$

and all agents choosing strategy D *if the inequality is reversed.*

Proof: See Appendix.

As we might expect, the minimum number of commodities where indirect exchange can be supported as the unique long run outcome is three – when the double coincidence problem first arises.

We then have the following result on long run equilibria in this case.

Corollary 2.4 *Given an sequence of economies* $E(l)$ *with fixed* m *and* β *satisfying lemma 2.3, there exists an* l^* *such that for all economies with* $l > l^*$ *the unique long-run equilibrium has all agents using strategy* A.

Proof: See Appendix.

Thus with a little noise we are able to characterize the degree of complexity required for a monetary equilibrium to be eventually achieved, and almost always observed. We might wonder if there is anything meaningful in the result, which is a limit result. We would like to know how long before the monetary equilibria is reached if the economy did not begin there. For a reasonably complex economy, this time might be so long as to make the result largely irrelevant. Agents will switch to using tokens from the direct barter equilibria whenever there are $\frac{n(l^2-l)}{(l-2)(1-m)+1}$ or more simultaneous mutations among the $n(l^2-l)$ members of the population. Thus although the number of mutations needed increases with l, as a fraction of the population it decreases.

Lemma 2.5 *For an economy with* $0 < m < 1$, *and discount factor* $0 < \beta < 1$, *the expected wait to reach the "All A" equilibrium decreases as the number of commodities becomes large.*

$$\lim_{l\to\infty} W(\epsilon) = 0$$

Proof: Using the normal approximation to the binomial,

$$W = \left[1 - Z\left(n(l^2-l)\frac{1-\epsilon((l-2)(1-m)+1)}{((l-2)(1-m)+1)\sqrt{n(l^2-l)\epsilon(1-\epsilon)}}\right)\right]^{-1}.$$

The argument of Z goes to $-\infty$ as l gets large. ∎

We have already warned about taking the efficiency comparisons too literally given the way tokens displace consumable commodities in the model economy. It is still worth noting that efficiency and and convergence do not coincide in this model. In

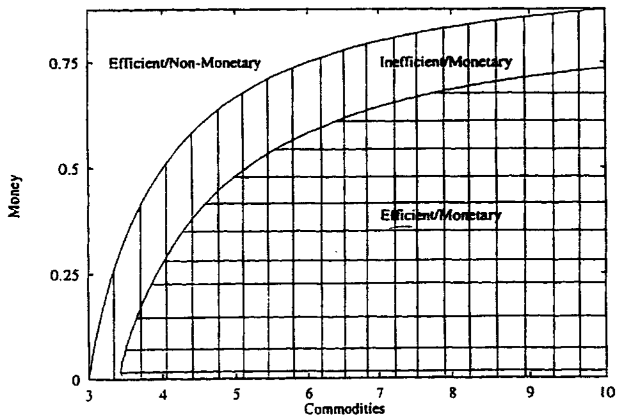

Figure 2.5: Long-Run Equilibrium vs. Efficiency (Agg. info)

fact, as figure 2.5 shows, the stochastically stable (long-run) equilibria is the monetary equilibria before there are enough commodities for it to be efficient. [9]

2.3.2 Case 2: Disaggregated Information

The ability to provide a clear analytical result in the example above hinged on the the assumption that agents have very limited information. We would like to be able to examine situations where agents have better information because it might not be clear to what degree our results rely on the information structure.

We now consider the situation where agents learn the exact distribution across

[9]This may be related to risk dominance, but since in this model expected payoff is not a linear function of population strategy, the right notion of risk dominance is not the usual one.

types of the strategies used. With the more detailed information agents are given. we can no longer exploit symmetry to simplify the complicated dynamic system. The value functions are complicated.[10]

Although we are unable to prove a theorem analogous the Theorem 2.3 for this case, we can compare the results of simulations. We include for comparison in the simulations a third information structure. We were able to prove an analogous result for this structure, where the information given agents about the strategy choices is aggregated only by endowment type (ie what fraction of each "endowment sector" accepted tokens) in the previous round. We do not include the analysis of this case because the algebra doesn't doesn't provide much intuition.[11] Figures 2.6 and 2.7 are representative, they shows that convergence not only still occurs - it is considerably faster. We summarize in Table 2.3.2 the effect of the number of commodities on convergence time in simulations.[12] This leads to the question of why convergence is

[10]Given a expectation of population strategies S, they are

$$V(D|S) = \sum_{r=1}^{\infty} \beta^r q_r^\theta(c|e) \prod_{s=1}^{r-1} \left(1 - q_s^\theta(c|e)\right),$$

and

$$V(A|S) = \sum_{r=1}^{\infty} \beta^r \left[q_r^\theta(c|e) + q_r^\theta(m|e) v_r^m\right] \prod_{s=1}^{r-1} \left(1 - q_s^\theta(c|e) - q_s^\theta(m|e)\right),$$

given that

$$v_r^m = \sum_{s=r+1}^{\infty} \beta^{s-r} q_s^\theta(c|m) \prod_{w=r+1}^{s-1} \left(1 - q_w^\theta(c|m)\right).$$

[11]A fourth case, where information is aggregate by consumption type results in similar properties to the completely aggregated case.

[12]This is especially rough for cases where the passage time was long. We ran simulations with 4000 switch opportunities in each case. Thus there were few or no switches towards the top of the table. The parameters used in Figure 2.7 were: $l = 6$, $m = .15$, $\epsilon = .13$, and $n = 3$. The only change in Figure 2.7 is that $\epsilon = .03$ and $l = 7$. For Table the parameters were 2.3.2 $m = .25$ and $\epsilon = .07$.

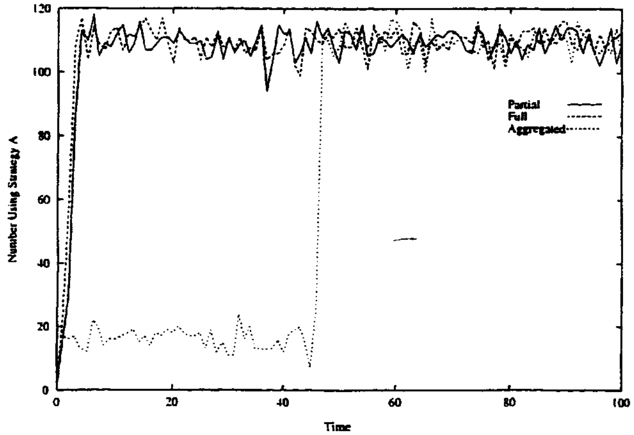

Figure 2.6: Eventual Convergence of Three Information Structures

faster when there is more information.

The agents' payoffs are principally affected by the strategies of agents with whom they could trade (and only secondarily by those who trade with those agents). Specifically the agents who matter most to an agent of type θ are those who affect $q_r^\theta(c|e)$ or $q_r^\theta(c|m)$. Better information about producers' trade strategy gives agents the ability to react to those agents whose strategy matters. The model has a particular "location" structure (in terms of commodities) and only with the detailed information can agents exploit it. In the static context the extra information gives the model more equilibria (ones like those discussed in section two).

These additional equilibria are the source of the increase in convergence speed.

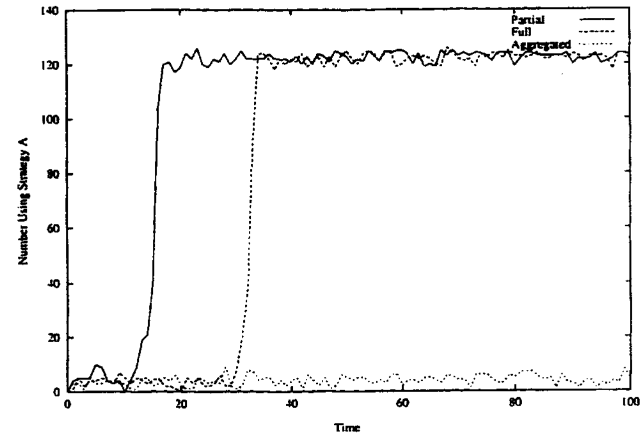

Figure 2.7: Rapid Convergence of Finer Information Models

	Information Structure		
l	Complete	Partial	Aggregate
3	?	?	?
4	?	?	?
5	136.3	2000.5	?
6	11.8	8.8	?
7	6.7	8.9	?
8	4.3	5.3	?
9	3.6	4.5	?
10	2.9	3.5	1000.3
11	2.6	2.9	232.9
12	2.5	2.5	30.3
13	2.3	2.3	14.1
14	2.1	2.3	6.3
15	2.2	2.3	2.3

Table 2.1: Average First Passage Time (Barter to Monetary)

A mutation moves the state of the system, but if it was insufficient to escape the basin of attraction of the equilibria, the system returns to the initial equilibria. With only two equilibria, as in the first example, each has a large basin of attraction. For a mutation to cause a response it must be large enough to convince the entire population to immediately switch and this occurs only with a low probability.

The dynamic system is like a hill, the two extreme equilibria are its two sides. Jump to the top and you can roll to the bottom of the other side, but if you jump short you roll back to where you began and have to try again. As the number of commodities increases the hill gets higher. In the simple example, the theorem the says that the two sides don't grow at the same rate[13] and it gets relatively harder to make the switch from the monetary side compared to the return trip. But it may be a long time before a successful jump is made.

The increased number of equilibria is like the addition of ledges to the hill, where one can rest between short jumps. Not only that, but the steps are only the non-monetary side, making it relatively easy to get to the monetary equilibria and very hard to get out.[14]

We can make a perturbation to the model that helps show why this is so. So that the decision for an agent would be much simplified, suppose that each agent is

[13]Stretching the metaphor, on the sides of the hill the relative elevations of the valleys are changing.

[14]The intuition for this is already available from research on local interaction. From the switch of some locale (which is of relatively high probability), the effect quickly spreads to the rest of the economy. Part of this intuition has been formalized in Ellison. He defines a modified transition cost that takes account of the lower cost of a path that passes through stable sets. His bound would still not capture fully the increased speed of convergence here because his bound is unable to account for the possibility that mutations out of intermediate stable sets may be more likely to move us closer rather than further away.

only given one trading opportunity after his choice of strategy and the random token opportunity. All he would have to look at is whether the number of agents he expects to be holding his consumption good and using strategy A is greater than the number of agents with whom he can make a double coincidence trade. Thus any agent's best response is to use A if previously

$$(1-m)\sum_{\dot{\theta}\in c^{\theta}} a_{\dot{\theta}} > (1-m)a^{\theta'} + d^{\theta'}.$$

A mutation necessary for this to be satisfied for some agent (and thus column of agents) becomes more likely as the number of commodities increase. Furthermore the number of columns that must switch for the entire population to follow and to switch does not increase - so this also becomes increasingly likely. Finally there are a number of smaller mutations of agents switching from D to A (such as to equilibria mentioned in section 2), that although leading directly to all monetary exchange, make the mutation necessary to complete the transition much more likely.

On the other hand for the switch from all A to all D the number of mutations required is larger in economies with more commodities. This type of mutation because an very low probability occurrence.

2.4 Conclusion

We have shown that regardless of initial conditions, in an economy with a diverse enough collection of commodities, a population of agents will develop a social convention that is characterized by a monetary system of exchange. Furthermore this convention will emerge in finite time, unaided by any explicit coordination mechanism such as a government.

We have seen in this model that the level of detail in the information that agents have about others' trade strategies is critical in speeding up the transition. In particular, this allows relatively small behavioral changes within a group of agents, who are all endowed with (or produce) the same commodity, to have a large affect on the population. This is interesting both because it speeds the transition to monetary equilibria and because the effect is asymmetric, similarly sized behavioral changes in a group of agents who consume the same commodity will not spread. Further work with a model that allows sectoral asymmetries in information may show that visible trade, such as takes place in markets, can not only assist a monetary system, but also result in particular market structures.

Our model points to the importance of the relationship between specialization and exchange. Diverse commodity demands increase the the double coincidence of wants problem and this impels the economy to develop a monetary system. Thus it is clear that the presence of money, or more precisely the degree of difficulty in finding trade opportunities, should be closely related to the degree of specialization. Further work, in a model where agents choose their consumption and production goods may shed light on the manner in which a money encourages specialization and specialization encourages monetary exchange.

The main lesson to be drawn is that intermediated exchange is a robust institu-

tion. Despite difficulties imposed by the model – inability to explicitly coordinate and the displacement of real resources by the intermediate good – in an economy with a reasonably diverse set of commodities, intermediated exchange is the unique stochastically stable state over direct barter. It remains for future research to examine the relative robustness of particular forms of intermediated exchange.

Chapter 3

Aggregate Risk, Idiosyncratic Risk, and Rates of Return with Incomplete Markets

3.1 Introduction

This research examines the determination of rates of return in an incomplete markets environment. In particular we look at the interaction of idiosyncratic and aggregate uncertainty in determining rates of return. We examine the manner in which the value and rate of return of an asset, held in part due to self-insurance motives, is affected by the sizes and mix of common risks, ie. flood damage, and individual risks, ie. variations in crop productivity.

Similarly, we may imagine that there could be important differences in undertaking

similar projects in small countries versus large ones. Casual empiricism suggests that interest rates are higher in financially isolated small countries than in large ones. It is not clear to what extent this is due to the individual risks being undertaken or to capital costs in the economy. To put the question more precisely, does the importance of idiosyncratic risk in determining rates of return depend in some manner on the size of aggregate risk?

Insurance against fluctuations must be arranged through trade with other individuals, either with insurers or by buying and selling assets to alleviate risk. If there are complete markets only aggregate fluctuations matter - individuals can insure against everything else. In the absence of insurance (incomplete markets), agents need to use assets to self-insure. If there is no aggregate uncertainty the terms on which they can do this are unchanging - only their own financial position may matter (a credit limit for example). When there is also aggregate uncertainty the ease with which persons in the economy can self-insure depend on how large their insurance needs are and how the values of the assets with which they must insure are correlated with their need to buy and sell them. Thus the rate of return of insuring assets will depend in a critical way on the subtleties of the relationship between aggregate in idiosyncratic risk in the economy.

In the model economy agents are subject to endowment risk and can only insure against them by accumulating and decumulating real asset balances. The real assets have a risky return. We examine the properties of parameterizations of some of the simplest incomplete markets economies that are interesting: those with two agents. one asset, and one consumption good. By varying the process generating agents' endowments and the consumption payoff of the asset we will examine the role of individual and aggregate uncertainty in determining the equilibrium behavior of the asset's real rate of return.

We look for Markov equilibria in these economies. It is difficult to guarantee existence of Markov equilibria, but it has properties, such as regularity over time and simplicity, that make it an attractive equilibrium notion. The approach in this paper is to follow Levine (1993) is look at approximate equilibria, with an ϵ-approximating model that guarantees existence of strong Markov equilibria. This is Levine's notion of *trembling invisible hand equilibrium.*

Early asset-pricing models, such as Lucas (1978) relied on shocks to a representative agent (aggregate fluctuations) to generate dynamics. The representative agent is reasonable in a complete markets environment because agents can perfectly insure against the effects of idiosyncratic shocks. Aggregate consumption behaves as if it were chosen by some single large single agent.

More recently research attempting to address the equity premium puzzle, posed in Mehra and Prescott (1985), has examined the role of uninsurable risk (typically calibrated to labor income and unemployment probabilities).[1] To avoid technical complications and to focus on idiosyncratic risk, aggregate uncertainty is not present – thus interest rates are constant. Although this research has found a heterogeneity

[1] See for example Aiyagari and Gertler (1991) and Heaton and Lucas (1992).

effect on interest rates, it has been too small to account for a significant portion of the equity premium puzzle. Although asking how the equity premium is effected by idiosyncratic and aggregate uncertainty is within the scope of the present research, here we focus on the effect of the interaction in determining a single rate of return.

In a paper closely related to this one, Den Haan (1996) addresses the issue of how heterogeneity (indexed by the number of types of agents) affects short-term interest rates and wealth dispersion in the presence of aggregate uncertainty. Solving the nonlinear models in this research is typically difficult and there is no existence result for Markov equilibria of the type Den Haan looks for. Thus an additional method of characterizing economies of this type serves as a useful check on the reliability of the parameterized expectations method used there. An advantage of our approach is that we use an equilibrium notion that guarantees existence of Markov equilibria, so it is possible that we can to find our Markov equilibria where the more familiar sort fail to exist. An additional differences between this paper and Den Haan's are that we consider agents that are asymmetric as well as being heterogenous and that agents smooth consumption using a risky asset rather than a riskless one-period bond.

We find that rates of return generally increase with increases in either aggregate or idiosyncratic risk. For fixed levels of idiosyncratic risk, average rates of return are increasing in the level of aggregate risk because although individual wealth fluctuations are the same, the fraction of the time when both agents are very rich (very poor) simultaneously increases. Because of risk aversion, both agents desire to save more at the same time (save less at the same time) and this lowers (raises) the rate of return. The volatility of rates of return is greater and mean is increased.

For fixed levels of aggregate risk, rates of return decrease with increasing idiosyncratic risk. This is because the increased individual fluctuations increases the need for agents with endowment risk to use the asset to smooth their consumption. As a result they pay more for the asset in their wealthy state and get less in their poor state.

3.2 The Model

We begin with a general description of an exchange economy with real assets. As this is a special case of the more general model in Levine (1993), we will keep the notation much the same. There are two agents, $a \in \{1, 2\}$, one consumption good, and one real asset. Time is discrete and doubly infinite, $t \in \{\ldots, -1, 0, 1, \ldots\}$.

There is an exogenous state variable η_t that takes values from a finite set I. The exogenous state follows a strictly positive Markov process, so transition probabilities can be written as $\pi(\eta_{t+1}|\eta_t) > 0$. At t the history $\eta_t, \eta_{t-1}, \ldots$ is common knowledge.

Each agent a has a strictly positive endowment at t, $\bar{x}^a(\eta_t) > 0$. Period consumption is x_t^a. Asset holdings at the beginning of t for agent a are given by y_t^a. The real (consumption good) return of the asset at the beginning of period t is given by $R(\eta_t) > 0$. The aggregate stock of the asset is fixed and positive, given by $\bar{y}$.

Let p_t be the price of the consumption good and q_t be the price of the asset (one

price can be normalized, but we leave it for now). The ex post rate of return on the asset is

$$i_t = \frac{1}{q_{t-1}} [p_t R(\eta_t) + q_t],$$

and the ex ante return is found by taking the expectation with respect to information at $t - 1$.

Agents choose their consumption profile and portfolios to maximize their lifetime utility function,

$$U_t^a = \mathrm{E}_t \left[\sum_{\tau=t}^{\infty} \delta^{\tau-t} \frac{(x_\tau^a)^{1-\alpha}}{1-\alpha} \right], \tag{3.2.1}$$

subject to a budget constraint,

$$p_t(x_t^a - \bar{x}^a(\eta_t) - R(\eta_t) y_t^a) + q_t(y_{t+1}^a - y_t^a) \leq 0. \tag{3.2.2}$$

We also have the following market clearing conditions: social feasibility of asset holdings;

$$\sum_{a\in\{1,2\}} y_t^a = \bar{y}, \tag{3.2.3}$$

and social feasibility of consumption;

$$\sum_{a\in\{1,2\}} x_t^a = \bar{x}(\eta_t) + R(\eta_t)\bar{y}. \tag{3.2.4}$$

3.2.1 Equilibrium

Generally, an equilibrium of this economy will map the complete history of the state (endogenous state variables (portfolios) and the exogenous state variable) into sequences of prices, portfolios, and consumptions. However, with exogenous uncertainty a Markov process and the simple asset structure, it seems like equilibria could have a simple structure where the dynamics of the economy depend only on the current distribution of assets and the current exogenous state. In a Markov equilibrium agents in the economy have a good sense of how likely different things are to happen in the future. In strong Markov equilibria this is a sensible description - they have this knowledge because the connection between events today is simple and the same as it has been in the past.

The notion of equilibrium with these attractive properties has been formalized as *strong Markov equilibrium*. This consists of *strong Markov plans* for the economy (functions $p(y_t, \eta_t)$, $q(y_t, \eta_t)$, $x(y_t, \eta_t)$, and $y(y_t, \eta_t)$) that give tomorrow's allocations and prices in the economy as functions of today's exogenous and endogenous state variables. To be an equilibrium these plans must satisfy individual optimality and both individual and social feasibility constraints.

Strong Markov equilibria are only known to exist under certain conditions. General existence results for Markov equilibria in incomplete market economies with both idiosyncratic and aggregate uncertainty require an augmented state space that includes player continuation values as in Duffie, Geanakoplos, Mas-Colell, and McLennan (1994).[2] Adding these continuation values to the state is not attractive as they

[2]This is not part of the state in Den Haan, so although there is recourse to an existence result given in Magill and Quinzii (1994), this result says nothing about stationarity. Thus Den Haan's approximation of discretized stationary distributions which may or may not be equilibria would troubling unless they are verified to be equilibria.

are not economically relevant to other agents.

This problem is addressed in Levine (1993) and a weaker equilibrium concept is introduced. This notion is *ϵ-trembling hand equilibrium*. Strong Markov equilibrium supposes that the knowledge of causal links in the economy is exact. Here the knowledge of causal links is fuzzy because the links themselves are made fuzzy. Given agents' plans, the "market" chooses a price and an allocation, not just a price (at which traders buy and sell as much as they like). Furthermore a "tremble" is incorporated into this market process. This means that agents may not receive their desired bundle, however equilibrium requires that they can expect to receive a bundle that is close to the bundle they wanted. Effectively, discretization is made part of the model, the tremble keeps actual portfolios on a finite grid and the trembles (along with the grid itself) are chosen so that agents choices are ϵ-close to their expected portfolio. Trembling invisible hand equilibria are guaranteed to exist, though they are not necessarily unique.

Because there is now a distinction between the intended portfolio and the realized portfolio, additional notation for end of period asset holdings, z_t^a, is needed.

For trembling invisible hand equilibrium we make use of the notion of *random Markov plans* in place of the strong Markov plans in strong Markov equilibria. Rather than requiring $y_{t+1} = z(y_t, \eta_t)$, that current distribution of assets is determined completely by the previous distribution of assets and the state, in a random Markov plan the *probability* of different distributions of assets is determined by the previous state.

formally

$$\text{Prob}(y_{t+1}, \eta_{t+1} | \eta_t, \eta_{t-1}, \ldots, y_t, y_{t-1}, \ldots) = \pi(y_{t+1}, \eta_{t+1} | y_t, \eta_t).$$

An ϵ-trembling hand equilibrium is a random Markov plan the satisfies the feasibility constraints and cannot be improved upon by more than ϵ.

3.2.2 Equilibria of a Fictitious Economy

A method of calculating equilibria is provided by looking for equilibria of a related *fictitious economy*. Equilibria plans in this economy have equivalents in our original model and an exact equilibrium here with small transfer payments here corresponds to a small ϵ in the trembling hand equilibria there.[3]

Let S be the set of states, giving the composition of the grid of asset distributions with the set of exogenous states. For now take as given π, a strictly positive transition matrix on S, and let π_s be the unique stationary probabilities associated with these state transition probabilities. The agents' objective in the fictitious economy is to maximize

$$U^a = \sum_{s \in S} \pi_s \frac{(x_s^a)^{1-\alpha}}{1-\alpha}. \tag{3.2.5}$$

There is a budget constraint for each state, these are defined so that agents have the same incentives to hold assets as in the original economy. Associated with π is a *backward* transition matrix μ, so that $\mu_{s\sigma}$ gives the probability at σ of having previously been in state s. Agents choose their desired portfolios in each state z_s^a and

[3]See Levine (1993) for complete details.

their resources from asset holdings is weighted by the probability of having previously been in the different states. For each state that might have previously been, the agent has a weighted average of the assets he chooses to hold and to guarantee positive endowments a small fraction of the aggregate assets. Thus the budget constraint for agent a in state s is

$$p_s(x_s^a - \bar{x}^a(\eta_s)) + q_s z_s^a - r_s(1-\gamma) \sum_{\sigma \in S} \mu_{\sigma s} z_\sigma^a + \gamma \bar{y}/A \leq 0. \tag{3.2.6}$$

From this we find first order conditions for each agent: in each state agents equate the marginal rate of substitution of each consumption good:

$$\pi_s \frac{\partial u^a(x_s^a, \eta_s)}{\partial x_s^a} - \lambda_s p_s \geq 0 \qquad x_s^a \geq 0 \tag{3.2.7}$$

and each asset that agents hold they hold in quantity to equate marginal value in consumption today to discounted expected value in consumption tomorrow:

$$\delta \sum_{\sigma \in S} \lambda_\sigma r_\sigma \mu_{s\sigma} - \lambda_s q_s \geq 0 \qquad z_s^a \geq 0. \tag{3.2.8}$$

An equilibrium of this fictitious economy solves (3.2.6), (3.2.7), and (3.2.8) (as well as complementary slackness conditions) and the feasibility conditions (3.2.3) and (3.2.4).

3.3 Computational Approach

The computational objective achieved in two parts. The first is the solution of a system of the above simultaneous equations giving first-order conditions from agent optimization, agent feasibility, and market clearing. In this part of the problem agents take as given the "fuzziness" in the relationship between their choice of portfolio and its actual composition. Because the economy is already finite, no discrete approximation is necessary, as opposed to most other approaches. We use a global Newton method to find the solution to the system of equations. [4]

The transition probabilities that compose this fuzziness are then updated based on the distances of the choices from the grid so that states in which the chosen portfolios are closest to the grid asset portfolios are given the most weight in the transition matrix (while satisfying the restriction implied by the exogenous state transition matrix). A fixed point argument used in Levine (1993) shows that this process will result in an ϵ-trembling hand equilibrium so that the expected effect of the trembles is arbitrarily small for each agent individually and in sum.

3.4 Calibrating the economy

Now that we have a method of solving for these equilibria, we turn to characterizing the properties of some economies. One approach would be to select parameters implied by micro studies of the economy and then check how the numbers produced by the model match aggregate data. However in this paper we are interested in checking for a qualitative effect, so we choose the parameters somewhat arbitrarily.

[4] See Press, Teukolsky, Vetterling, and Flannery (1992).

3.4.1 An Example

To understand the economy more fully, let's look in detail at an equilibrium of a particular (semi-arbitrary) parameterization.

Parameter		Value
Discount factor	δ	0.98
Coefficient of Risk Aversion	α	1.50
Stock of Real Asset	$\bar{y}$	1.0

Here agents' endowments are uncorrelated and the asset has a certain return. One agent has a risk-free endowment while the other faces endowment risk that is correlated with the return of a risky asset. There are four states as follows (add in asset payoff):

	Endowment of Agents		Asset Return
State	$\bar{x}^1$	$\bar{x}^2$	R
1	1.0	0.85	0.05
2	1.0	1.15	0.10
3	1.0	0.85	0.10
4	1.0	1.15	0.15

The obvious choice for the exogenous state transition probability matrix is:

$$\pi(\eta_t = i | \eta_{t-1} = j) = .25 \qquad i, j \in \{1, 2, 3, 4\}.$$

We fix a grid of allocations and look for an ϵ-trembling hand equilibrium. This is close to the grid.

The solution of the economy has agent 1 holding on average 70.1% of the asset and consuming on average 50.9% of the available consumption good. The expected return on a unit of asset is 3.7%. As is clear from figure 3.3, this fluctuates substantially with the exogenous state. Less clear from the figures is that although the agents consumption fluctuates, their fraction of total consumption is constant.

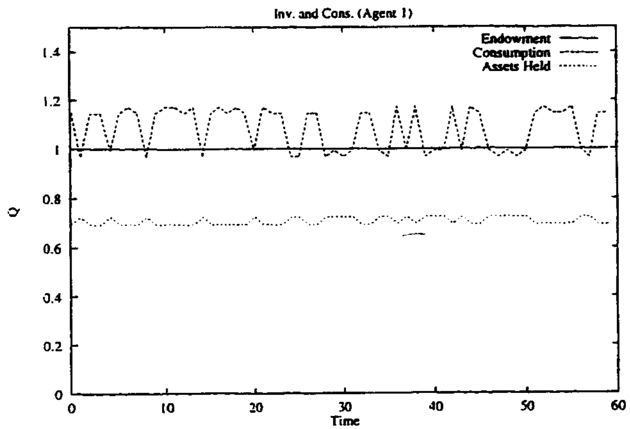

Figure 3.1: Agent 1 - Endowment, Portfolio, and Consumption

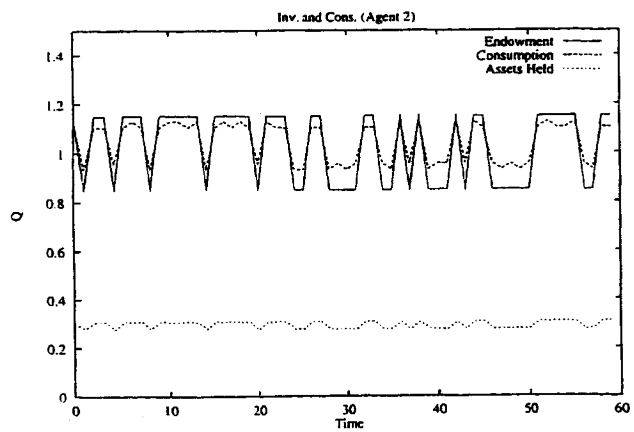

Figure 3.2: Agent 2 - Endowment, Portfolio, and Consumption

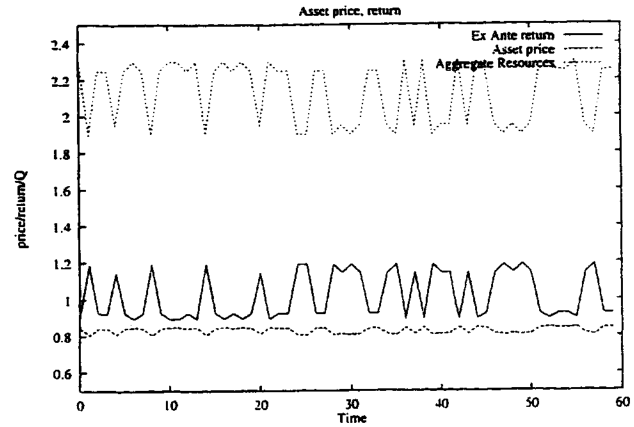

Figure 3.3: Some Economy-Wide Data

3.4.2 The Effect of Idiosyncratic and Aggregate Uncertainty on Rates of Return

We will vary the amount of uncertainty in the economy by changing the productivity of the asset and the endowment of agents. In all cases we keep the mean consumption pay out of one unit of asset at 0.1 and the mean individual endowment of the consumption good at 1.0, as in the example economy.

First consider a baseline economy with no uncertainty were agents' endowments and the asset's real return are always 1 and .1, respectively. As expected, agents split ownership of the asset, on which there is a real return of 2.041%. First we would like to take this starting position and evaluate the effect on rates of return of increasing the amount of aggregate uncertainty without changing the amount of idiosyncratic endowment variability. There are two ways of doing this within our two agent model: increase the variability of the asset's return or if there is variability in individual endowments we can change the correlation between agents endowment patterns. We take the former approach because it is easier to implement and it seems to result in numerically robust results for a wide range of asset return patterns.

In figure 3.4, the line for the first set of economies (no idiosyncratic) shows the rate of return for economies with assets that have standard deviation of their productivity ranging from 0.0 to 0.495. In the figure we put the implied aggregate endowment variability on the x-axis.[5] Despite the constant mean return, agents' risk aversion implies that as the variability of the asset increases (and the variability of the aggregate endowment) the equilibrium rate of return must increase to compensate the holder for the risk he must take.

Now let's add variability to agents' endowments. Agents prefer smooth consumption but the only way they can insure against endowment uncertainty is to accumulate and decumulate the asset. This gives an additional motive for holding the asset, so as we see the this shifts the curves in the figure down: an insurance motive for holding assets raises their price, or equivalently, lowers the return on the assets.

If we want to view the effect of increasing idiosyncratic uncertainty while holding

[5]Note that our lines do not all have the same range. The reason the lines in figure 3.4 do not necessarily start at zero is because here only one agent has idiosyncratic variability, thus even with no variability in the asset, there is still some minimum variation in the aggregate endowment. The reason that economies were not computed so that all lines stretch equally far is more troubling. The simplistic non-linear equation solver we used failed past this point. More robust computational methods should help.

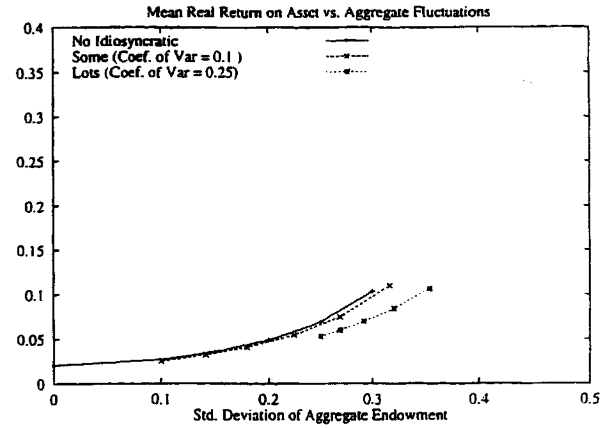

Figure 3.4: Expected Return vs. Std. Dev. Aggregate Uncertainty

aggregate variability fixed we have a bit of a challenge. Aggregate uncertainty is composed of asset productivity variability and endowment variability. The former is easy to hold fixed, but the latter closely ties together individual and aggregate uncertainty. Thus in figure 3.5 we do not hold the aggregate endowment variability fixed but instead the ratio of aggregate endowment variability to individual endowment variability. This is fixed by the correlation of the endowments of the two agents. The productivity of the asset is certain. In the lines shown in the figure there are respectively, perfectly negatively correlated individual endowments (no aggregate), uncorrelated (some), and perfectly correlated (lots). Along each line we increase the size of the fluctuations of individual endowments. When there is no aggregate en-

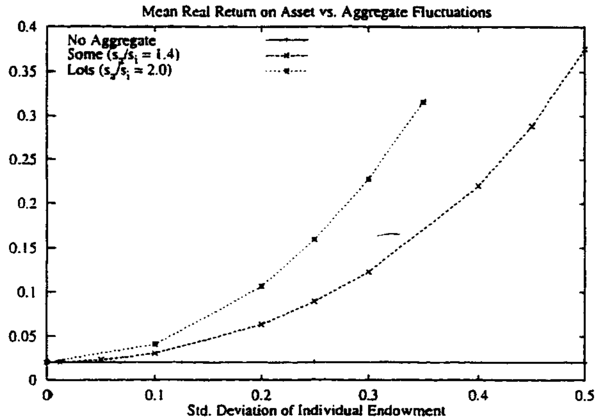

Figure 3.5: Expected Return vs. Std. Dev. Idiosyncratic Uncertainty

dowment uncertainty, the asset is able to insure individual risks by some quantity being passed back and forth, the average return is the same as when there was no uncertainty at all.

With increased aggregate uncertainty, there are more likely to be agents who are simultaneously wealthy or simultaneously poor. Because agents want to save more when they are richer, this results in increased fluctuation in asset prices (and returns). The curvature of agents utility functions leads to the average rate of return being higher when the variation in their wealth is greater.

3.5 Conclusion

We investigate the interaction of idiosyncratic and aggregate uncertainty in determining rates of return. We find that there is an interaction. That ceteris paribus increasing aggregate uncertainty increases rates of return and increasing the amount idiosyncratic uncertainty decreases rates of return.

It is difficult to say whether the results are quantitatively important without undertaking further analysis that calibrates the model economy to real world micro-features.

We should note that our approach, while providing a useful check on other strategies of characterizing incomplete markets equilibria, does suffer from a curse of dimensionality problem. As the number of agents is increased the computational demand will grow exponentially.

Appendix A

Proofs of Lemmas and Theorems

A.1 Proofs for Chapter 2

Proof of Lemma 2.1: Denote the conditional values of agents by $v_r(a)$, $v_r(m)$, and $v_r(d)$ (we suppress the type temporarily for notational simplicity). Thus whether an agent chooses strategy A or not depends on whether or not

$$v_0(a) \geq v_0(d).$$

The value are given recursively by

$$v_r(a) = \beta\left[q_r(c|e) + q_r(m|e)v_{r+1}(m) + (1 - q_r(m|e) - q_r(c|e))v_{r+1}(a)\right]$$

$$v_r(d) = \beta\left[q_r(c|e) + (1 - q_r(c|e))v_{r+1}(d)\right]$$

$$v_r(m) = \beta\left[q_r(c|m) + (1 - q_r(c|m))v_r(m)\right]$$

Generally, we see that

$$\begin{aligned} v_r(a) - v_r(d) &= \beta\{q_r(m|e)\left[v_{r+1}(m) - v_{r+1}(a)\right] + \\ &\quad (1 - q_r(c|e))\left[v_{r+1}(a) - v_{r+1}(d)\right]\}. \end{aligned}$$

Thus A has a higher payoff than D if

$$q_r(m|e)\left\{v_{r+1}(m) - v_{r+1}(a)\right\} \geq 0 \text{ for all } r,$$

and is strictly greater for at least one r, (and D has the higher payoff if the inequality is reversed). Since $q_r(m|e) \geq 0$ for all r, we need for all r,

$$v_r(m) - v_r(a) \geq 0.$$

This is given by the following, and if for all r, $q_r(c|m) \geq q_r(c|e)$ we have the inequality below.

$$\begin{aligned} &= \beta\{q_r(c|m)(1 - v_{r+1}(m)) - q_r(c|e)(1 - v_{r+1}(a)) \\ &\quad +(1 - q_r(m|e))\left[v_{r+1}(m) - v_{r+1}(a)\right]\} \\ &\geq \beta\left[(1 - q_r(m|e) - q_r(c|e))(v_{r+1}(m) - v_{r+1}(a))\right] \\ &\stackrel{r\to\infty}{\longrightarrow} 0 \end{aligned}$$

Finally, $q_r(c|m) \geq q_r(c|e)$ if and only if

$$\sum_{k=1}^{t} a_r^{(j,k)} \geq a_r^{(j,i)} + d_r^{(j,i)},$$

equivalently if

$$\sum_{\substack{k=1 \\ k\neq i}}^{t} a_r^{(j,k)} \geq d_r^{(j,i)}.$$

The second half of the proof follows a parallel argument, basically only reversing the inequalities. ■

Proof of Lemma 2.3: Let K be the number of agents that used strategy A in the previous period. Then agents believe that $\frac{K}{l^2-l}$ agents of each type will use A. Thus,

$$d_0 = n - \frac{K}{l^2 - l},$$

$$a_0 = (1 - m)\frac{K}{l^2 - l},$$

and

$$m_0 = m\frac{K}{l^2 - l}.$$

It is then trivial (using the definitions of $q(c|e)$, $q(m|e)$, and $q(c|m)$) that these are constant for all rounds. Using Lemma 2.1, the condition under which an agent (and thus all agents) are willing to choose strategy A is

$$(l-2)(1-m)\frac{K}{(l^2-l)} \geq n - \frac{K}{l^2-l}.$$

or, rearranging to solve for K,

$$K \geq \frac{n(l^2-l)}{(l-2)(1-m)+1}.$$

Agents choose D if the inequality is reversed. All states other than "All A" or "All D" are transient. The dynamic path will consist of mutations that either carry us into the basin of the other stable equilibria, or just return back to the initial equilibria. A state is stochastically stable, or a long run equilibria if the limit of the unconditional probability of being at that state goes to one as $\epsilon \to \infty$. An equivalent condition is

$$\lim_{\epsilon \to \infty} \frac{P(\text{Switch to } D|\text{Basin of } A)}{P(\text{Switch to } A|\text{Basin of } D)} = 0$$

The switch to A and the switch to D take K and $N-K$ simultanous mutations respectively, so this is equivalent to

$$\lim_{\epsilon \to \infty} \frac{\epsilon^{N-K}}{\epsilon^K} = 0.$$

The condition for this is $N - 2K > 0$, and so "All A" is the long run equilibria if

$$l > \frac{1}{1-m} + 2.$$

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